MONTE CARLO MODELING OF EXOSPHERIC BODIES: MERCURY

Gerald R. Smith
Kitt Peak National Observatory, Tucson, Arizona 85726

D. E. Shemansky[1]
Lunar and Planetary Laboratory, University of Arizona, Tucson, Arizona 85721

A. Lyle Broadfoot and L. Wallace
Kitt Peak National Observatory, Tucson, Arizona 85726

Abstract. Previous model calculations of the helium exospheres of Mercury and the moon have been based on a regime in which each helium atom impact with the surface results in the selection of a new particle chosen from a given source distribution. The particular velocity space distribution in the source particles was chosen with the implied intent that the resulting atmosphere would be barometric under ideal conditions. In effect, two particle source distributions have been used in the published calculations, which we describe here as Maxwell-Boltzmann (M-B), and Maxwell-Boltzmann flux (M-B-F). In the instances in which the atmospheric distribution has been calculated for regions above the surface, the resulting model atmosphere represents a mixture of the M-B and M-B-F sources. We suggest that none of the published exospheric calculations for the two bodies represent atmospheres produced by a barometric source of particles. Although a barometric source of particles cannot be justified in terms of surface physics, an exosphere produced by such a source is a valuable point of reference for calculations based on more realistic conditions. According to the analysis presented below, the appropriate source distribution should be M-B-F if the particles in the distribution are to be treated as components of flux. Monte Carlo calculations with an M-B-F source are compared with Mariner 10 ultraviolet spectrometer data. The comparison suggests that present models are incapable of fitting the observed Mercury exosphere.

1. Introduction

Several studies have been made of Mercury's helium exosphere [Hartle et al., 1973, 1975; Hodges, 1974]. Two basic assumptions were made in the model used in these studies. These assumptions were that the surface is 'saturated' and that on impact with the surface the helium atom is trapped and within an indeterminate time another helium atom is re-emitted with a Maxwellian velocity distribution appropriate to the local temperature. Broadfoot et al. [1976] concluded from calculations based on this model that satisfactory agreement with the observations of the Mariner 10 ultraviolet spectrometer experiment could not be obtained.

In order to study the interaction with the surface we have developed a Monte Carlo program designed to determine the distribution with altitude as well as the global distribution of density at the surface in a single operation. We audit densities at concentric spheres located at various height intervals. Thus densities are audited directly, whereas previous investigators [Hodges, 1973b, 1974; Hartle et al., 1975] audited the number of impacts at the surface and used these impact counts to obtain the global distribution of surface number densities by relations tied to the Maxwellian velocity distribution. Their method of choosing source particles (for determining the impact count) was to select the three rectangular components of velocity space from one-dimensional Boltzmann distributions, in a Monte Carlo calculation. This provides what we describe as a Maxwell-Boltzmann (M-B) distribution as a source flux. We point out below that this (M-B) source does not provide the barometric exosphere that one would normally expect from an ideal atmospheric exobase [cf. Brinkmann, 1970] (see the note added in proof at the end of this paper). In order to obtain a barometric atmosphere under ideal conditions we require what is described in this article as a Maxwell-Boltzmann flux (M-B-F) distribution. The atmospheric modeling process in the cases cited above actually mixes the M-B and M-B-F sources in the production of an atmosphere. That is, the process of obtaining the number density at the surface from impact counts involved the use of the M-B source in the Monte Carlo calculation. The next step in the process, that of calculating the atmospheric distribution [Hartle et al., 1975], was equivalent to assuming an M-B-F source, since the height distribution was assumed to be barometric with the base density derived from the previous Monte Carlo calculation. This therefore constituted an internal inconsistency in the modeling process.

In the present work the source distribution is applied consistently throughout the modeling process by necessity, since the method is a single-stage operation, as opposed to the two-stage calculations described above. We encounter a fundamental difficulty in attempting to determine which of the two source distributions is the appropriate one for the model calculation, for neither can be physically justified by our present understanding of the physics of surface interactions. However, we suggest that an atmosphere calculated with a barometric source distribution is a useful point of reference for more realistic future exospheric models.

2. Source Distributions Applied To The Calculation Of Model Exospheres

The velocity and angular distributions of the source particles are of extreme importance to the

[1]Present address: Kitt Peak National Observatory, Tucson, Arizona 85726.

Paper number 8A0403.
0148-0227/78/088A-0403$01.00

determination of the exospheric distribution. This is especially true in the case of the models under discussion here, since in these computational regimes each impact with the surface results in the selection of a new particle from the source distribution. We wish to avoid the details of gas-surface interaction as much as possible at this point in the discussion and simply concentrate on the nature of the source distribution that has been applied in the model calculations for the moon and Mercury. The discussion is confined in this manner in order to point out an inconsistency within the computational method that tends to invalidate most of the published models. The difficulty appears to arise in those model calculations that depend at least partially on the Monte Carlo method.

Under normal circumstances when one has an atmosphere forming the base of the exosphere, atoms are assumed to be ejected into the exosphere according to a Maxwellian velocity distribution determined by the thermosphere below. The source distribution for the calculation of an exosphere on this basis has been the subject of controversy, culminating in the Brinkmann [1970] (cf. Chamberlain and Campbell [1967]) work. Brinkmann pointed out that the appropriate normalized source flux distribution has the form

$$f_3' \, dv = 1/2(MkT)^2 \, v^3 \exp -(Mv^2/2kT) \, dv \qquad (1)$$

where the quantities have their usual meaning. We describe (1) in this article as an M-B-F distribution. An M-B-F source produces a Maxwell-Boltzmann velocity distribution in the exosphere [Feynmann et al., 1963], and as Chamberlain [1963] has pointed out, the Maxwellian distribution holds even in the collision-free region; the atmosphere is barometric, provided one can neglect the thermal loss of particles.

When the base of the exosphere is the solid surface of the body, a new and more difficult problem is introduced. In this case it is not at all clear what angular distribution one should apply to the source without going into the detail of the surface interaction and the geometric condition of the surface itself. Moreover, there is no particular reason why the surface interaction should produce a Maxwellian distribution in the gas [Shemansky and Broadfoot, 1977].

Hodges [1974] approached the problem of determining the global surface number density by selecting the three rectangular components from a Maxwell-Boltzmann distribution as a source for a Monte Carlo calculation. The surface number density was then determined through transformation of the impact counts with the application of the Maxwellian velocity distribution. The impact counts and particles in these calculations were treated as components of flux (see note added in proof). This is where the inconsistency enters, since the selection of velocity components in Hodges' method results in a flux for the Monte Carlo calculation of the following form:

$$f_2' \, dv = 4\pi(M/2\pi kT)^{3/2} \, v^2 \exp -(Mv^2/2kT) \, dv \qquad (2)$$

Equation (2) is described here as an M-B distribution. A flux of this form will not provide a barometric atmosphere if the particles are chosen as components of flux, and one then cannot estimate number densities from impact counts by assuming a Maxwellian gas. Hartle et al. [1975] appear to have followed Hodges' [1974] method in applying an M-B-F source to the calculation of an exosphere utilizing a global surface number density distribution inconsistent with a barometric source. Hodges' [1973a] analytic calculations, as we demonstrate below, are consistent with an atmosphere produced by a barometric source, and apparently for the reasons cited above do not agree with the later Hodges [1973b, 1974] and Hartle et al. [1975] Monte Carlo calculations. We will show in the following text that a consistently applied Monte Carlo calculation for a nonuniform atmosphere with an M-B-F source tends to agree more closely with the Hodges [1973a] analytic solution.

3. The Monte Carlo Program

In applying the Monte Carlo method, extreme care must be taken to insure that the results are consistent with the physical processes. We have taken the direct approach and sampled the physical state as directly as possible.

The time spent in traversing a horizontal slab is

$$\tau = \frac{\ell}{v \cos \chi} \qquad (3)$$

where ℓ is the slab thickness, χ is the angle to the normal, and v is the speed of the particle at height r. If ℓ is differentially small, then we can reasonably ignore grazing effects and write the relative density as

$$\langle n(r) \rangle = (R/r)^2 \, \Sigma\tau_i \qquad (4)$$

where the sum is over all particles which cross the surface located at height r. Audit zones at the surfaces were established having equal areas and located symmetrically with respect to the equator [cf. Hodges, 1973b]. Audit zones at each altitude layer were directly above the surface zones and subtended the same solid angle. Thus the geometric factor $(R/r)^2$ in (4) must be applied. It is the relative density because it can be scaled by a constant over the whole planet depending on the strength of the sources and sinks.

Hodges [1973b] gives the appropriate equations and formulas in order to transform to the θ (colatitude) and ϕ (longitude) coordinates from the orbital system. The complete eliptical orbit theory was applied. Selection of new particles was made from a uniform global source. Thus each source particle had an equal probability of coming from any location on the planet. Since the particle audit is carried out in three dimensions, we use a fairly small number of grid areas.

The program has been verified in the following manner: (1) Several orbits were examined in detail. The intercept points of a particle were traced through the spherical layers from source point to impact point in order to determine that they all resided in a plane passing through the global center. Intercept angles to the layers were also verified. (2) The source normally used in exosphere modeling is an M-B-F distribution, and there is a substantial pool of computations for uniform atmospheres that one can use for comparison [Brinkmann, 1970; Chamberlain and Campbell,

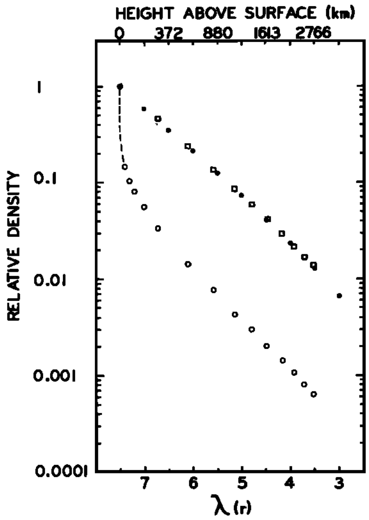

Fig. 1. Atmospheric number densities calculated in the present work in comparison with the Chamberlain [1963] data, normalized to 1 at the surface. The solid circles are Chamberlain's data for $\lambda_c = 7.5$. The squares are the Monte Carlo results for an M-B-F source, and the open circles are the Monte Carlo results for an M-B source. The applied parameters approach the case of helium on Mercury above the subsolar surface.

1967]. The program was operated for this purpose with appropriate parameters to compare with the Chamberlain [1963], $\lambda_c = 7.5$, calculations. Figure 1 demonstrates the compatibility of the results. (3) A nonuniform atmosphere was calculated with parameters appropriate for comparison with one of the Quessette [1972] calculations for hydrogen on the earth. The Quessette case 4 was chosen, in which surface temperature varied over the planet sinusoidally and symmetrically from subsolar point to antisolar point with T_{max} = 960°K and T_{min} = 720°K. An antisolar/subsolar density ratio at the base of 1.87 was obtained, compared with the Quessette value of 1.89. (4) Comparison was made with the Hodges [1973a] analytic results given by his equation (17). Calculations for Mercury for two different surface temperature variations are shown in Figures 2 and 3. The agreement is quite satisfactory in view of the statistics of the Monte Carlo results. The temperature distributions used in the calculation of Figures 2 and 3 are shown in Figure 4. The solid curves are from Chase et al. [1976]. The Chase et al. equatorial data were extrapolated from the equatorial region to the poles by assuming the variation in temperature in the polar regions on the dark side to be similar to that at the equator. We note that in these cases the antisolar/subsolar surface number density ratios are 140 and 150, compared with a value of 200 in the Hartle et al. [1975] calculation. This represents the difference in global mobility between the M-B-F and M-B source, as discussed below. The present calculations using an M-B source provide a ratio of 270, somewhat larger than the Hodges [1974] and Hartle et al. [1975] results. The reason for this discrepancy arises in the computational methods. The present work calculates the number density at the surface directly, using an M-B source. The Hodges [1974] and Hartle et al. [1975] number densities at the surface are obtained by applying a Maxwellian distribution to impact numbers that were calculated with an M-B source. An M-B source produces relatively greater numbers of low-energy particles at the surface as a function of decreasing temperature. The application of an M-B-F distribution to the calculation of number density from impact numbers thus tends to decrease the antisolar/subsolar number density ratio relative to the exact solution. Thus all of the results presented here can be reasonably explained in terms of the consequences of the computational methods.

4. Significance of the M-B and M-B-F Sources to Atmospheric Distribution

In order to demonstrate the effect of choosing one or other of the source fluxes for atmospheric altitude distribution, our program was operated for the case of a uniform atmosphere, in which temperature at the surface was constant around the globe. The results are shown in Figure 1, with normalization to unit number density at the surface. The M-B source clearly produces a distribution divergent from a barometric atmosphere, and the preponderance of slow particles is demonstrated in the sharp divergence from an exponential atmosphere near the surface. The calculations using an M-B-F source shown in the figure have

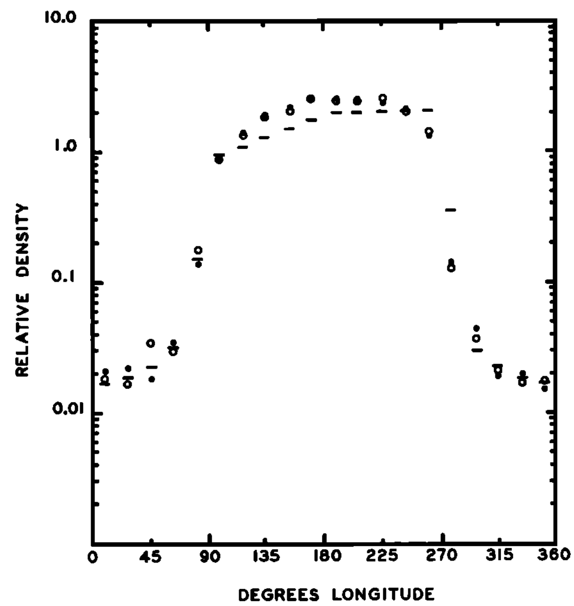

Fig. 2. Normalized Monte Carlo surface densities in the equatorial region of Mercury for helium. These densities have been normalized by the average density in the equatorial region around the planet. The surface temperature closely approximates the results of Chase et al. [1976] and is shown in Figure 4. The solid and open circles represent the auditing zones closest to the equator. One set is just above the equator; the other is just below. The total number of surface collisions was 400,000. The dashes represent the results of Hodges' [1973a] equation (17) for the same temperature. Thermal escape and photo-ionization loss are taken into account in the Monte Carlo calculation.

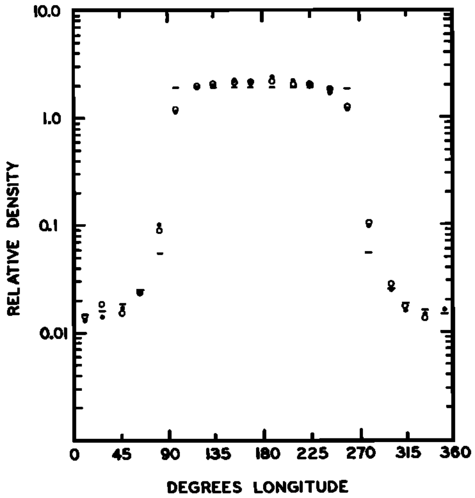

Fig. 3. Normalized Monte Carlo surface densities in the equatorial region of Mercury for helium. This figure is similar to Figure 2, except that the surface temperature distribution is the same as that of Hartle et al. [1975] (see Figure 4). The total number of surface collisions was 800,000.

been discussed earlier. The choice one makes for a source distribution is clearly an important one in terms of thermal loss rates and atmospheric altitude distribution. The consistent use of an M-B source thus produces a relative density a factor of 20 below a barometric (M-B-F) source above 250 km in this particular case. The classical Jeans escape factor thus cannot be applied. The M-B loss rate equivalent to Jeans escape is given by

$$\phi_s = \phi_{up}[\mathrm{erfc}\ (\lambda^{1/2}) + 2(\lambda/\pi)^{1/2}\ e^{-\lambda}] \qquad (5)$$

where ϕ_s is escape flux due to an M-B source and ϕ_{up} is the total upward flux.

The ratio of ϕ_s to ϕ_j (Jeans escape) is

$$\phi_s/\phi_j = \frac{\mathrm{erfc}(\lambda^{1/2}) + 2(\lambda/\pi)^{1/2}\ e^{-\lambda}}{(1 + \lambda)\ e^{-\lambda}} \qquad (6)$$

Where we assume ϕ_{up} to be equal in both cases. Now at $\lambda = 7.6$ (appropriate for Mercury at aphelion on subsolar point)

$$\phi_s/\phi_j = 1/2.60$$

or a loss rate of 2.60 times less than the barometric atmosphere. At T = 200°K the ratio is

$$\phi_s/\phi_j = 1/4.15$$

A more detailed discussion of (5) and (6) is provided as an appendix to this article.

Using this ratio in the equations given by Hodges [1974], one finds that the absolute densities must increase by some factor greater than 2.6 or by some average of the ratio ϕ_s/ϕ_j over the illuminated part of the planet.

5. Helium Model for Mercury

The Monte Carlo program was then applied to the case of helium on Mercury, using the expanded Chase et al. [1976] temperature distribution, and an M-B source distribution. The calculation allowed for both thermal escape and photo-ionization loss, as was the case of the M-B-F calculations shown in Figure 2. Densities were first audited only at the surface by using a single concentric audit sphere located at the surface. One characteristic of the application of an M-B as opposed to an M-B-F source is the relative difficulty in obtaining satisfactory statistics for the distribution estimates. This is due to relatively large density fluctuations caused by the large pool of slow atoms in the M-B distribution. Acceptably low statistical fluctuations were obtained for 10^7 surface impacts, compared with about 5×10^5 impacts for equivalent fluctuations for an M-B-F source. The antisolar/subsolar density ratio is about 270, 35% larger than the Hodges [1974] and Hartle et al. [1975] values. The surface number densities at the equator for this case are shown in Figure 5. In order to obtain an altitude distribution a second calculation was made with 14 audit spheres. The results of this calculation are shown in Figures 6 and 7, convolved with the instrumental function of the Mariner 10 UV spectrometer, and compared with observed Mercury data. The model is normalized to the limb data of Figure 6. The data off the subsolar limb obtained at 12,000 km instrument-planet range does not fit the model particularly well near the limb, and the data as a whole have a smaller slope over the observed altitude region. The model calculated with an M-B-F source, not shown here, tends to provide a more satisfactory fit to the data of Figure 6. The data shown in Figure 7 [cf. Broadfoot et al., 1976] were obtained at a range of 85,000 km. In this case the instrumental slit length was about the size of the planet diameter, and more of the global distribution of the atmosphere was integrated in the signal than the close range data. The data above the subsolar limb, in

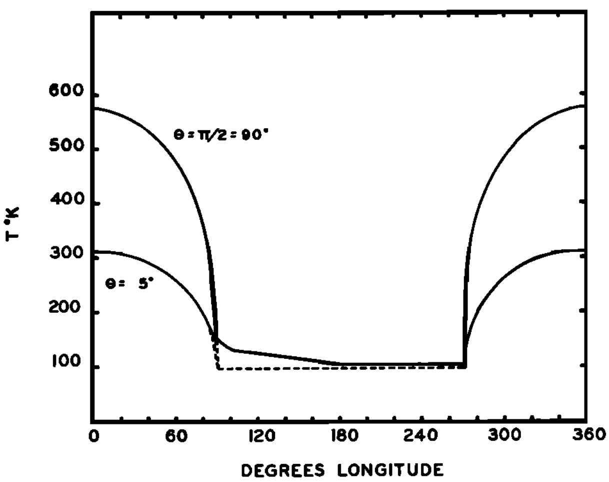

Fig. 4. Temperature profiles for Mercury. The solid curves are the temperature derived from Chase et al. [1976]. The dashed curves are the temperature profiles used by Hartle et al. [1975]. The dayside temperature was assumed in both cases to be $T = 575\ (\sin\theta\ \cos\phi)^{1/4}$ (°K).

contrast to Figure 6, now appear to have a larger slope than the model. The absolute intensities above the subsolar limb at the two ranges differ by factors of 1.5-2, suggesting a divergence of global distribution between model and data. The data obtained near the terminator in Figure 7 also demonstrate a significant phase difference with the model. The terminator peaks of model and data are 600 km apart in this figure. Model calculations using an M-B-F source provide a less satisfactory fit to the data at 85,000-km range. Thus the model with either source does not consistently fit the data [cf. Broadfoot et al., 1976]. The application of an M-B source raises the subsolar surface number density estimate by a factor of ~ 5 relative to the value obtained with an M-B-F source. The estimated thermal escape rate (2.6×10^{22} s^{-1}) for the M-B model is about a factor of 10 below that for the M-B-F Mercury model. Thermal loss in this case is about equal to the photo-ionization loss.

The two sources that have been applied to Mercury and lunar exosphere modeling thus present widely differing implications for atmospheric evolution. However, as we point out below, a physical argument cannot be developed for the acceptability of either distribution.

6. Importance of Gas-Surface Interactions

In the above discussion we have demonstrated that our Monte Carlo computational method can reproduce established work on uniform and nonuniform exospheres with the application of the appropriate input parameters. We have pointed out that previous calculations involving the application of the Monte Carlo method to exospheric bodies have been internally inconsistent in that a mixture of source distributions have been applied to the production of model atmospheres. The use of an M-B source in the Monte Carlo modeling of surface density distributions is reminiscent of the controversy that developed earlier, which led to puzzling discrepancies among published theories of exospheric distribution and estimates of thermal

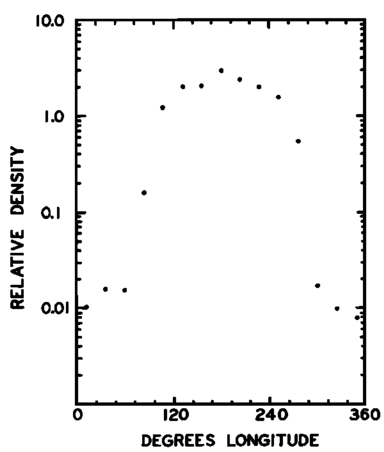

Fig. 5. The surface density of helium on Mercury derived from the M-B source model. The number of surface collisions was 10^7. The audit zones just above and below the equator were averaged, followed by a two-point average to obtain the final result as shown.

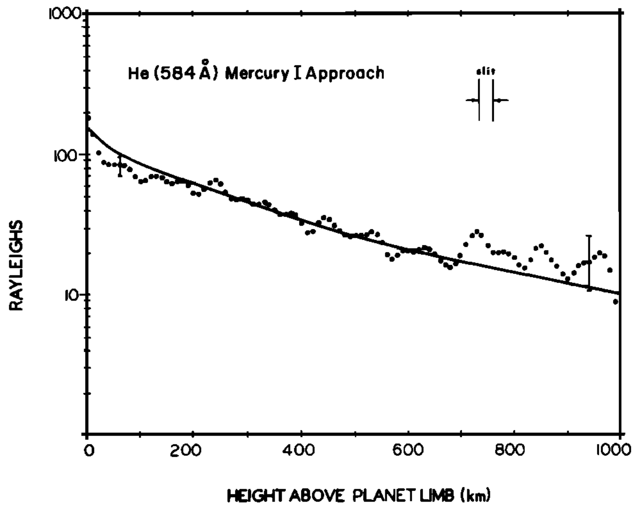

Fig. 6. Mariner 10 helium data and M-B source model off the limb of Mercury at a range of 12,000 km. The solid circles are the observational data, and the solid line is the model convolved with the instrumental function. The three points near the limb contain signal from surface scattering. The data have been corrected for interstellar background. Model and data are spatially integrated densities.

escape rates [Chamberlain and Campbell, 1967; Brinkmann, 1970; Chamberlain and Smith, 1971]. Lew and Venkateswaren [1965], for example, selected their parameters from an M-B rather than an M-B-F distribution [Brinkmann, 1970]. The two source distributions also have a differing angular distribution, as Brinkmann has pointed out, the M-B-F source having a median angle of 45° as opposed to a value of 60° for the M-B source.

The question now arises as to what distribution is the correct one for an exosphere interfaced with a solid surface. There is little doubt as to which distribution is the appropriate one for an exosphere interfaced with an atmosphere, since in that case the physics of the scattering process [Brinkmann, 1970] has been examined in detail. No comparable consideration has entered the lunar or Mercury modeling process, and the nature of the source with necessary minimum alterations appears to have been borrowed from the earlier work. However, the interaction of a gas atom with a solid surface is distinctly different from the processes that occur at an atmospheric interface. This has been discussed recently at some length by Shemansky and Broadfoot [1977], and we simply point out the general nature of the problem in the following text.

The application of an M-B distribution as a source of atmospheric particles is an assumption based on uncertain physical considerations. For modeling purposes the nature of the gas-surface interaction has been described in the following way: (1) The surface is assumed to be saturated. This is defined by a process in which an impacting atom must be accompanied by the release of another atom at the impact site. (2) The subsequent ejection is chosen from an M-B source distribution. Thus each impact with the surface must be accompanied by a new source particle. The physical implication of this is that the impacting particle must reside for some indefinite, unspecified long

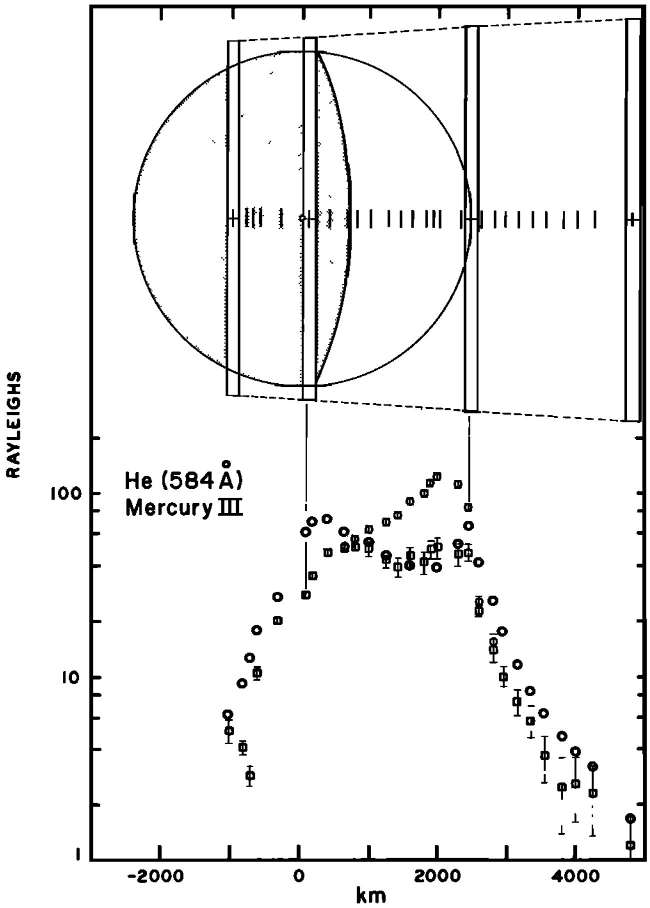

Fig. 7. Mariner 10 helium data. The light circles represent data obtained during a drift across Mercury at an average range of 85,000 km on the incoming pass (evening side). The squares represent observational data corrected for planet surface albedo. The error bars represent statistical variations at that point. The heavy circles are the model results normalized by the comparison of model with the subsolar limb data shown in Figure 6.

period of time on the surface. This is where the basis for the exospheric models involved in this discussion encounters serious difficulty, for in order to define the nature of the atmospheric source function in terms of physics at the surface, one must specify an average residence time. This fundamental quantity must be determined before a discussion of the nature of the surface interaction can begin. If the residence time is extremely short, the probability for an exchange of energy between surface and gas atom will be commensurately small, and the gas atom will leave the surface with a memory of its preimpact energy. Thus under this condition we cannot have a new source particle chosen from an M-B distribution. If the residence time is very long, coupling to the surface must be strong, since interaction times with surfaces are typically extremely short in comparison with homogeneous collision times. Quantum physics must apply to this physical scale for light particles, and binding energy will be in the chemical scale of magnitudes. A saturated surface on the moon or Mercury demands heats of adsorption in the range 10^4 cal/mol to 10^5 cal/mol [Shemansky and Broadfoot, 1977]. In this extreme the gas atom is immobilized on the surface in a potential well invariably possessing an activation energy. An atmospheric source particle making its appearance under these conditions would certainly not possess the characteristics of a Maxwellian distribution. Binding energies between these two extremes would by no means guarantee a Maxwellian distribution. Coupling to the surface is quantized in energy levels with separations and populations which depend on the nature of the gas atom-surface coupling. Basically, what we are saying is that we do not have a system in detailed balance, since the exosphere has no influence on energy distribution in the surface, and in this case there is certainly no reason to believe that statistical equilibrium will approximate an M-B-F source distribution.

7. Conclusions

1. Those model calculations of the exospheres of the moon and Mercury that have utilized the Monte Carlo method [Hodges, 1973b, 1974; Hartle et al., 1975] have applied two different source energy distributions. The mechanism of control of the energy distribution of the exospheric gas in these calculations places the source energy distribution in the position of sole determinant. The chosen source energy distribution therefore places any discussion of evolutionary development of the exosphere in direct dependence on the argument for the choice. A physical argument for the use of either source is not forthcoming. One of the sources (M-B-F) under ideal conditions produces a barometric exosphere. This is the source that is normally used in the calculation of exospheres interfaced with an atmosphere. The other source (M-B) produces a nonbarometric exosphere with a much reduced thermal escape rate if the particles are treated as components of flux. Some of the model calculations cited used a mixture of the two sources, suggesting a mistake in the application of the Monte Carlo method.

The choice of an M-B or M-B-F source is not easy to defend with physical arguments. The nature of the source in the models depends on the average residence time of the gas atom on the surface. This quantity has never been specified in order to form a basis for discussion of gas surface coupling.

A discussion of exospheric evolutionary processes must involve the physics of gas-surface coupling [Shemansky and Broadfoot, 1977].

2. We are unable to obtain a consistent fit to the Mercury data with either of the sources discussed above. The use of the M-B and M-B-F sources illustrates the critical dependence of loss rates and global atmospheric distribution on the energy configuration of the source, established by the mechanics of the present models. Although the barometric source is not a realistic one, the resulting model atmosphere is valuable as a comparative distribution for more advanced model calculations.

Appendix: Calculation of Escape Fluxes in the Monte Carlo Method

In the Monte Carlo calculation presented above we have used two different distributions of source particles. We give here a discussion of the escape flux for each distribution having relevance to the validity of (5) and (6).

In the Monte Carlo process the source distribution is directly translated into a relative source flux for the atmosphere. Thus the total upward flux at the surface is proportional to the total number of upward going particles in the Monte Carlo process; that is,

$$\phi_{up} \propto J_o \quad \text{(A1)}$$

where ϕ_{up} is the total upward flux and J_o is the total number of particles going up in the Monte Carlo computation. Each source particle is therefore considered to be a component of upward flux in this and the other computational methods discussed in this article (see the note added in proof). We choose particles from a velocity distribution f(v) defined as a barometric, M-B-F source, such that

$$\int f(v)\, dv = 1 \quad \text{(A2)}$$

The upward flux is then written

$$\phi_{up}^{f}(v) = \frac{J_o}{4\pi R^2} \int_v^{\infty} f(v)\, dv \quad \text{(A3)}$$

The distribution f(v) is selected at the planet surface, and if we establish some fictitious source volume below the surface, the velocity distribution of particles in that volume must be g(v), the Maxwellian volume distribution, M-B. This is the source distribution chosen at the planet surface in the earlier published Monte Carlo calculations, such that

$$\int g(v)\, dv = 1 \quad \text{(A4)}$$

Since the particles have the same dimensions and are treated in the same manner in every case, we must write the flux equation

$$\phi_{up}^{g}(v) = \frac{J_o}{4\pi R^2} \int_v^{\infty} g(v)\, dv \quad \text{(A5)}$$

It can be shown that the two distributions f(v) and g(v) are related by the equation

$$f(v)/v = g(v)/\bar{v}_g \quad \text{(A6)}$$

where $\bar{v}_g$ is the mean velocity in the volume, defined by

$$\bar{v}_g = \int v\, g(v)\, dv \quad \text{(A7)}$$

The total number of escaping particles in each case derived from (A3) and (A5) is given by

$$\psi^f = J_o\, [\lambda + 1]\, e^{-\lambda} \quad \text{(A8)}$$

and

$$\psi^g = J_o\, [\text{erfc}\, (\lambda^{1/2}) + 2(\lambda/\pi)^{1/2}\, e^{-\lambda}] \quad \text{(A9)}$$

Given two atmospheres generated by each source, in which equal numbers of particles J_o are selected, then

$$\frac{\psi^g}{\psi^f} = \frac{[\text{erfc}\, (\lambda^{1/2}) + 2(\lambda/\pi)^{1/2}\, e^{-\lambda}]}{[\lambda + 1]\, e^{-\lambda}} \quad \text{(A10)}$$

The local number densities in the two cases must obviously be quite different, since the average velocities differ for a given upward flux.

Note added in proof. Since this paper was submitted two articles have been published on the subject of exospheric models of Mercury [Curtis and Hartle, 1978] and the moon [Hodges, 1977], and we have undergone considerable discussion on the subject with one of the referees of this article, R. R. Hodges. The Curtis and Hartle work follows the mechanics of the earlier work according to our understanding. The Hodges [1977] article about the moon, which introduces an interesting loss process, applies a computational method which differs fundamentally in the interpretation of the calculated quantities. Hodges chooses particles from an M-B source distribution but interprets impact counts as a measure of number density at the surface rather than as a measure of flux. The suggestion was that the present [Hodges, 1977] computational method applying an M-B source, and the method presented in this article applying an M-B-F source, may be equivalent. This is certainly true in the case of a body with uniform surface temperature. However, Hodges now agrees that his 1977 method contains a hidden variable scale factor in the general case and amounts to a non-ergotic transformation of the Monte Carlo particles. Furthermore our discussions have led to agreement that the choice of Monte Carlo particles in the method presented in this article must be made from an M-B-F source if a barometric atmosphere is to be produced in the idealized case. An M-B source distribution as applied in the earlier Monte Carlo calculations is not a barometric source and does not provide Jeans escape factor.

Acknowledgments. We wish to thank R. R. Hodges for enlightening discussions of the various Monte Carlo computational methods. The Kitt Peak National Observatory is operated by the Association of Universities for Research in Astronomy, Inc., under contract with the National Science Foundation. This work was supported by the Jet Propulsion Laboratory, California Institute of Technology, under NASA contract NAS 7-100.

The Editor thanks J. W. Chamberlain and R. R. Hodges for their assistance in evaluating this paper.

References

Brinkmann, R. T., Departures from Jeans' escape rate for H and He in the earth's atmosphere, Planet. Space Sci., 18, 449, 1970.

Broadfoot, A. L., D. E. Shemansky, and S. Kumar, Mariner 10: Mercury atmosphere, Geophys. Res. Lett., 3, 577, 1976.

Chamberlain, J. W., Planetary coronae and atmospheric evaporation, Planet. Space Sci., 11, 901, 1963.

Chamberlain, J. W., and F. J. Campbell, Rate of evaporation of a non-Maxwellian atmosphere, Astrophys. J., 149, 687, 1967.

Chamberlain, J. W., and G. R. Smith, Comments on the rate of evaporation of a non-Maxwellian atmosphere, Planet. Space Sci., 19, 675, 1971.

Chase, S. C., Jr., E. D. Miner, D. Morrison, G. Münch, and G. Neugebauer, Mariner 10 infrared radiometer results: Temperatures and thermal properties of the surface of Mercury, Icarus, 28, 565, 1976.

Curtis, S. A., and R. E. Hartle, Mercury's helium exosphere after Mariner 10's third encounter, J. Geophys. Res., 83, 1551, 1978.

Feynmann, R. P., R. B. Leighton, and M. Sands, The Feynmann Lectures on Physics, vol. 1, chaps. 39 and 40, Addison-Wesley, Reading, Mass., 1963.

Hartle, R. E., K. W. Ogilvie, and C. S. Wu, Neutral and ion-exospheres in the solar wind with applications to Mercury, Planet. Space Sci., 21, 2181, 1973.

Hartle, R. E., S. A. Curtis, and G. E. Thomas, Mercury's helium exosphere, J. Geophys. Res., 80, 3689, 1975.

Hodges, R. R., Jr., Differential equation of exospheric lateral transport and its application to terrestrial hydrogen, J. Geophys. Res., 78, 7340, 1973a.

Hodges, R. R., Jr., Helium and hydrogen in the lunar atmosphere, J. Geophys. Res., 78, 8055, 1973b.

Hodges, R. R., Jr., Model atmospheres for Mercury based on a lunar analogy, J. Geophys. Res., 79, 2881, 1974.

Hodges, R. R., Jr., Formation of the lunar helium cornoa and atmosphere, Proc. Lunar Sci. Conf. 8th, 537, 1977.

Lew, S. K., and S. V. Venkateswaren, A study of hydrogen diffusion in the earth's upper atmosphere near the critical level, J. Atmos. Sci., 22, 623, 1965.

Quessette, J. A., Atomic hydrogen densities at the exobase, J. Geophys. Res., 77, 2997, 1972.

Shemansky, D. E., and A. L. Broadfoot, Interaction of the surfaces of the moon and Mercury with their exospheric atmospheres, Rev. Geophys. Space Phys., 15, 491, 1977.

(Received July 15, 1977;
revised March 23, 1978;
accepted April 12, 1978.)