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True Satellite Ballistic Coefficient Determination for HASDM

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Abstract. The High Accuracy Satellite Drag Model (HASDM) requires true satellite ballistic coefficients (B) in order to estimate accurate atmospheric temperature/density corrections. Using satellite tracking data "true" B values were obtained for over 40 satellites that have been in orbit since 1970. Differential orbit corrections were computed from 1970 to 2001 every 3 days throughout the 31-year period for each satellite. The "true" B values were computed by averaging the nearly 3200 estimated B values obtained for each satellite. These "true" B values were validated by comparing the "true" B values of two spheres with theoretical values based on their known physical dimensions, and by comparing the "true" B values obtained for pairs of satellites having very similar size, shape, and mass. The estimated B variations for a number of satellites were then averaged over each year from 1970 to 2001, and compared with solar indices plotted for the last three solar cycles.

Introduction

For HASDM satellite drag is used to compute atmospheric density variations. Since the drag equations have the ballistic coefficient, B, and atmospheric density value factored together, the accurate computation of the density can be done only if the "true" ballistic coefficients of the satellites are know to within a small error. Thus, it was necessary to determine the "true" B values of as many HASDM satellites as possible to enable the HASDM estimation of accurate atmospheric density variations.

The "true" ballistic coefficient is given by $B = C_D A/M$, where C_D is the dimensionless drag coefficient, A is the cross-section area of the silhouette of the satellite viewed in the direction of satellite motion, and M is the mass of the satellite. The ballistic coefficient estimated during the orbit determination process is significantly different from the "true" B, due to errors in the atmospheric model providing neutral density. To differentiate this estimated value from the "true" ballistic coefficient B, we denote it as B′ which is given by:

$$B' \cong B\left[\int_0^{\Delta t} \rho \mathrm{v}^3 dt \Big/ \int_0^{\Delta t} \rho' \mathrm{v}^3 dt\right]$$

where ρ is the true density, ρ' is the model density, v is the velocity of the satellite, and dt is a differential element of time which is used to integrate over the fit span Δt. Strictly speaking, v^3 represents $\mathrm{v}_{rel}\left(\vec{\mathrm{v}}_{rel} \cdot \vec{\mathrm{v}}_{sat}\right)$, where $\vec{\mathrm{v}}_{rel}$ is the velocity of the satellite relative to the atmosphere and $\vec{\mathrm{v}}_{sat}$ is the velocity of the satellite relative to the inertial coordinate frame. v_{rel} is the magnitude of $\vec{\mathrm{v}}_{rel}$.

Differential Correction Orbit Fits

A batch least squares algorithm was used to fit each satellite's observational data to obtain B′ values. The batch differential correction (DC) data spans varied from 4 to 12 days depending upon the energy dissipation rate and the number of observations available per span. DCs were computed every 3 days starting from Jan 1, 1970 until Dec 31, 2000. Almost 100,000 radar and optical observations per satellite were processed over the 31-year period. Approximately 30 obs/DC were available for the 1970-1990 period, and approximately 60 obs/DC were available for the 1990-2000 period. Approximately 3200 estimated B′ values were obtained per satellite, and these values were used to obtain an average value, the "true" B value, over the 31-year time period. The perturbation model employed for the DC fits used a 48×48 truncation of the EGM-96 geopotential, lunar-solar third body gravitation, solid earth and ocean tides, and the Jacchia 70 atmospheric model for atmospheric drag. The 48 degree and order value was selected for the geopotential because it results in a significantly faster computer execution speed than the full 70 degree model, and the neglected higher degree and order terms (up to 70×70) produced less than a one percent standard deviation error in the ballistic coefficient due to aliasing.

A variable drag coefficient (C_D) approach was required for the processing. As previously reported (Bowman, 2001) the drag coefficient will vary with altitude based on the changing molecular constituents of the atmosphere (Afonso, 1985). Normally the C_D value is considered constant at a value of 2.2 for roughly spherical objects. However, when solar activity becomes low ($F_{10.7} < 80$) the dominant atmospheric species changes from atomic oxygen to helium at altitudes as low as 500 km. The drag coefficient with respect to atomic oxygen is approximately 2.2, while with respect to helium it approximates 2.8. Above 1500 km, when hydrogen becomes the dominant species, the C_D value is greater than 4.0. Therefore, the B value, the product of the drag coefficient and area-to-mass ratio, can vary by as much as 80% over a wide range of altitudes. If the C_D value does not vary based on molecular constituents then the error in C_D will be reflected in an error in B not attributed to just neutral density errors. Figure 1 shows the difference in the B′ coefficient from comparing B′ from orbit fits using a constant and then variable C_D value. Several satellites were used to compute B values every 3 days for the 31-year period, using first a constant C_D value of 2.2 for all the orbit fits, and then using a variable C_D value based on molecular species amounts. Figure 1 shows the differences (using variable C_D – using constant C_D) in the B′ value, for 3 different satellites with perigee heights varying from 510 km to 900 km. The differences in Figure 1 show that at altitudes of 900 km the B′ value can be in error from the C_D change by as much as 18% during low solar activity.

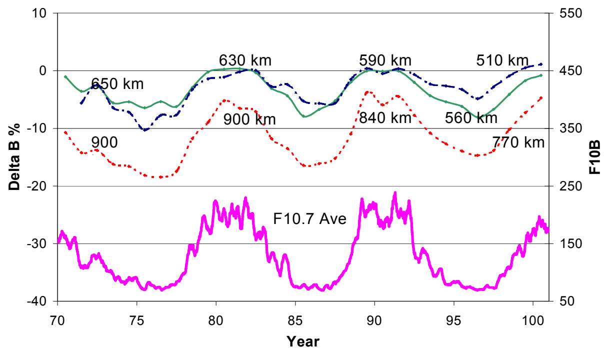

Figure 1. Change in B′ (variable C_D – constant C_D)

Figure 2 shows a history of B′ values over the 31-year period for satellite 00060, Explorer 8, which had a perigee height of approximately 400 km during the entire time period. Variations of as much as 50% with a standard deviation of 18% can be observed. Since the satellite is almost spherical in shape the variations are due, not to satellite area variability, but to unmodeled atmospheric density variations at 400 km altitude.

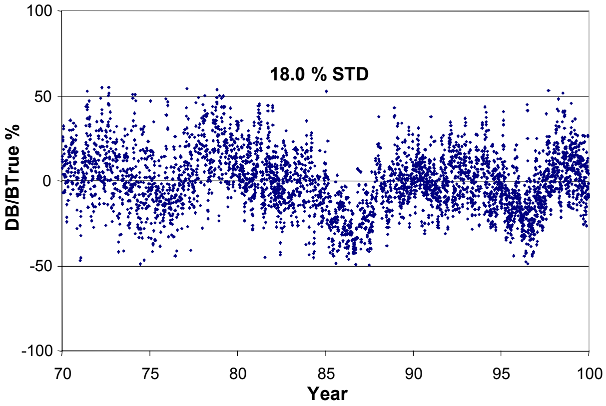

Figure 2. B′ variation (percent difference from “true” 31-year average) for satellite 00060, Explorer 8, with a perigee height of 400 km.

Validation of “True” B Values

The method of computing the “true” B value was validated by comparing the “true” B values of two spheres with theoretical values computed from their known physical dimensions. Vanguard 2 and AE-B were used in the analysis. Vanguard 2 was launched in 1959 with a perigee height of approximately 560 km, while AE-B was launched in 1966 with a perigee height of approximately 280 km. Vanguard 2 is still in orbit, while AE-B decayed in 1985. Average B values were computed, first for Vanguard 2 using 31 years of data, then for AE-B using 15 years of data from 1970 to 1985. From analysis of a number of other satellites’ “true” B values, it was determined that using the 1970-1985 time period produced an average B that was 2.5% higher than using the 1970-2000 period. Thus, the average B obtained for AE-B was reduced by 2.5% to obtain the “true” B value. The “true” B values for both satellites were then compared to the values computed from the C_DA/M expression. Table 1 shows the comparisons for a range of C_D values. It has been reported (Cook, 1965; Moe, 1996; Pardini, 1999) that the C_D value for spheres increases slightly from 2.2 to 2.4 for altitudes increasing from 250 to 500 km. These C_D values are estimated to have a standard error of 5%. Using the “true” B values and A/M computed from the dimensions, the C_D value can be computed that best fits the 31-year “true” B value. For Vanguard 2 the value is approximately 2.27, while for the lower altitude AE-B it is approximately 2.14. These values are in very good agreement with the expected C_D values plus or minus

5%. Thus, there does not appear to be a bias (to within 5%) in the “true” B values computed using the 1970-2000 time period.

SAT B 31 Yr	Q Ht Inc	A/M m2/kg	CD	CD*A/M	B31Yr/ CD*A/M
00011	560 km	0.02158	2.20	0.04748	3.3 %
Vanguard 2	32.9 deg		2.27	0.04899	0.1 %
0.04904			2.40	0.05179	-5.3 %
02183	280 km	0.00276	2.10	0.00580	2.1 %
AE-B	64.7 deg		2.14	0.00591	0.2 %
0.00592			2.20	0.00607	-2.5 %

Table 1. Comparisons of drag coefficients for two spheres with different perigee heights. Perigee height (Q Ht) and inclination (Inc) are included.

Another validation method consisted of comparing the “true” B values obtained for pairs of satellites with very similar size, shape, and mass. Three pairs of satellites were found that could be considered the same satellites in size, shape, and weight. Table 2 lists the pairs and the resulting “true” B values. The first two pairs consist of upper stage rocket bodies. The first pair was used to launch the Tiros 3 and 4 satellites into nearly identical orbits. Since both are solid fuel rockets, which burn until all the fuel is used, a good assumption is that the empty weights are the same. Also, since they were launched within a year of each other, it is a good assumption that they were produced as the same rocket body model. The “true” B values differ by less than 1% for this pair. The second pair of satellites are also upper stage solid rocket bodies, also used to launch satellites into nearly identical orbits. The difference of “true” B values is less then 0.1% for this case. Finally, the last pair considered is the Elektron 1 and 3 satellites, launched within 6 months of each other. They were both placed in very similar orbits with perigee and apogee heights of 400 km and 7100 km respectively. Even though the shape of these satellites is complex (cylinder with 6 paddles) the “true” B values compared to within 0.2%. Thus, the consistency of the “true” B values is readily demonstrated.

Below in Table 3 is a list of 31-year “true” B values for a number of selected satellites. The shape of the object plus additional orbital parameters are provided for each satellite.

Internat. Desig.	NORAD	Object	Qht km	Inc deg	B 31Yr	Diff. %
61-017B	165	Delta 1 R/B	740	47.9	0.05113	
62-002B	229	Delta 1 R/B	700	48.2	0.05157	0.9 %
65-072D	1583	Thor Altair R/B	649	98.6	0.04513	
66-026B	2129	Thor Altair R/B	634	98.6	0.04512	0.0 %
64-006A	746	Elektron 1	395	60.8	0.01603	
64-038A	829	Elektron 3	405	60.8	0.01600	-0.2 %

Table 2. Comparison of “true” B values for pairs of similar satellites.

1 Year Average B Values

Figures 3 and 4 show yearly averaged B' variations for satellite groups at 400 km perigee altitude in various inclination orbits, and then in high inclination orbits at various altitudes, respectively. The variations are a result of the unmodeled density variations using the Jacchia 1970 model atmosphere (Jacchia, 1970). The yearly B' values were obtained from a yearly averaging of the orbit fitted B' values. The plots show a remarkable consistency. The consistency at 400 km for different inclinations, and at high inclinations for different altitudes suggests that a global correction should account for the majority of the observed variations. Also of interest is the correlation of the yearly B' variations with the 90-day average solar flux $F_{10.7}$ index. The minimum yearly B' variation occurs during solar minimum, although the minimum B' variation does not level out as does the solar flux index. This indicates that the variation in the $F_{10.7}$ index does not account for all of the long term variation throughout the solar cycle.

2001 Atmospheric Density Variations

Figure 5 shows the results of the B' variations of a number of low inclination satellites for the first half of 2001 when the HASDM data was collected. The B' variations are plotted as a percent deviation from the “true” B value for each satellite. Also plotted are the solar flux indices $F_{10.7}$ and $a_{p.}$ The period from day 1 to day 75 shows very little solar variability even though the $F_{10.7}$ value is still moderately high (~150). During this period of time, the observed density increases more with height than is accounted for in the Jacchia model, as seen from the increase in the B' values with height. The higher the B' value over the “true” value, the higher the true atmospheric density relative to the model density. When solar activity variations increase after day 75, the atmospheric density appears to be more globally consistent with the predicted model since the B' variations for all the satellites are closer together.

However, there still appears to be a separation by height even during this period of time. Also of interest is the anti-correlation of the B′ variations with the $F_{10.7}$ variations. This indicates that the Jacchia model is consistently overcorrecting the density with respect to the solar flux index during the maxima, and under correcting during the minima. The HASDM model must account for, and remove, all these variations in order to be used as a global density correction model.

Conclusion

"True" B values can be obtained from averaging differential orbit corrected B values over the time period 1970-2000. These "true" B values show self-consistency when compared to identical satellites, and compare extremely well with values computed from dimensions of spherical satellites. The "true" B values can be estimated to be accurate to within 2-3% based on the above analysis.

Acknowledgment

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SAT NORAD	Internat. Desig.	Name	Object	True_B m2/kg	Q Ht 2001	Ap Ht 2001	Inc Deg
11	1959-001A	VANGUARD 2	Sphere	0.04904	560	3000	32.9
22	1959-009A	EXPLORER 7	Dbl Cone	0.02237	515	815	50.3
60	1960-014A	EXPLORER 8	Dbl Cone	0.02237	386	1145	49.9
63	1960-016A	TIROS 2	Cylinder	0.01428	526	580	48.5
107	1961-013A	EXPLORER 11	Cylinder	0.02068	485	1425	28.8
165	1961-017B	DELTA 1 R/B	Cylinder	0.05113	620	650	47.9
229	1962-002D	DELTA 1 R/B	Cylinder	0.05157	555	600	48.3
694	1963-047A	ATLAS 2 R/B	Cylinder	0.01678	465	1425	30.4
746	1964-006A	ELEKTRON 1	Cyl+paddles	0.01603	395	7125	60.8
829	1964-038A	ELEKTRON 3	Cyl+paddles	0.01600	405	7025	60.8
1583	1965-072D	THOR ALTAIR R/B	Cylinder	0.04513	614	890	98.3
1613	1965-078A	OV1-2	Cylinder	0.01669	405	2600	144.2
2129	1966-026B	THOR ALTAIR R/B	Cylinder	0.04512	507	605	98.2
2389	1966-070A	OV3-3	Octogon	0.01753	345	2880	81.4
2611	1966-111B	OV1-10	Cyl+booms	0.02322	504	560	93.4
4221	1969-097A	AZUR (GRS A)	Cone+Cyl	0.02111	375	1930	102.7

Table 3. "True" B values for selected satellites, with perigee height (Q Ht), apogee height (Ap Ht), and inclination (Inc) for the beginning of 2001.

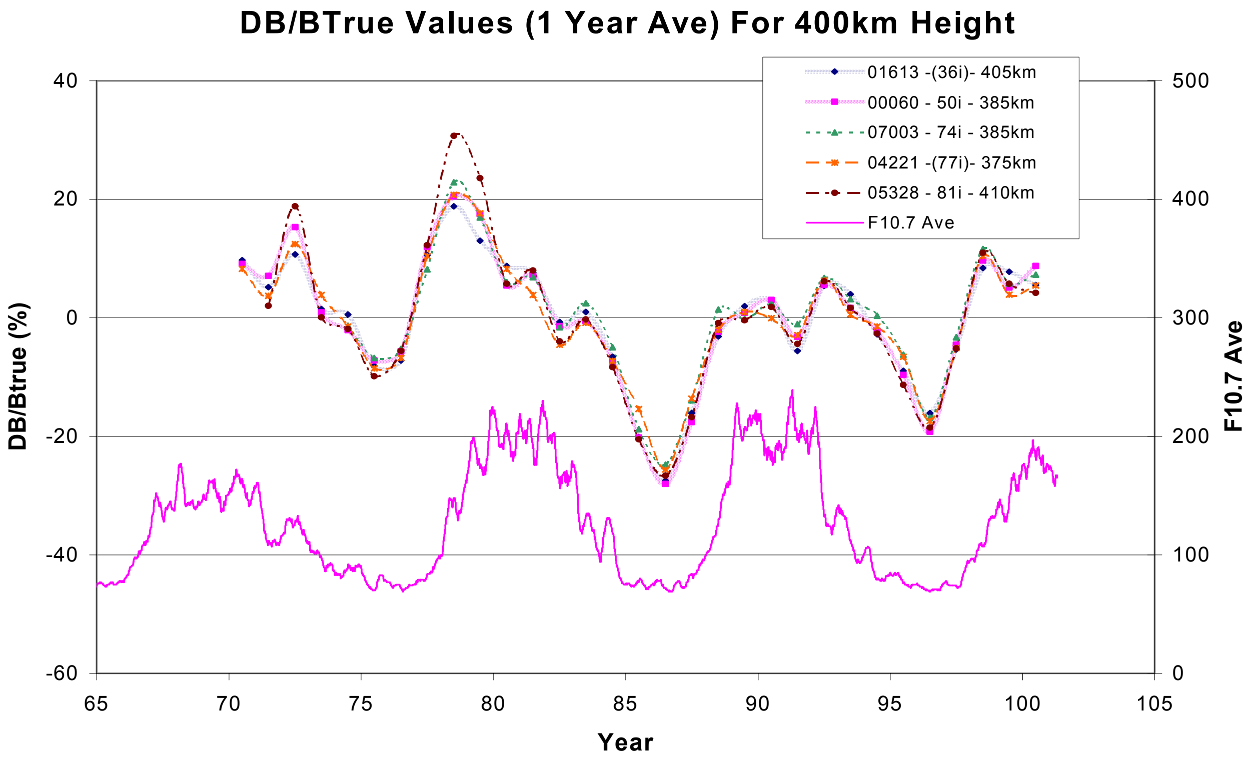

Figure 3. One year average B′ variations for satellites with perigee heights near 400 km. The inclination, i, and perigee height (km) are listed for each satellite. The parentheses around inclination represents a retrograde orbit.

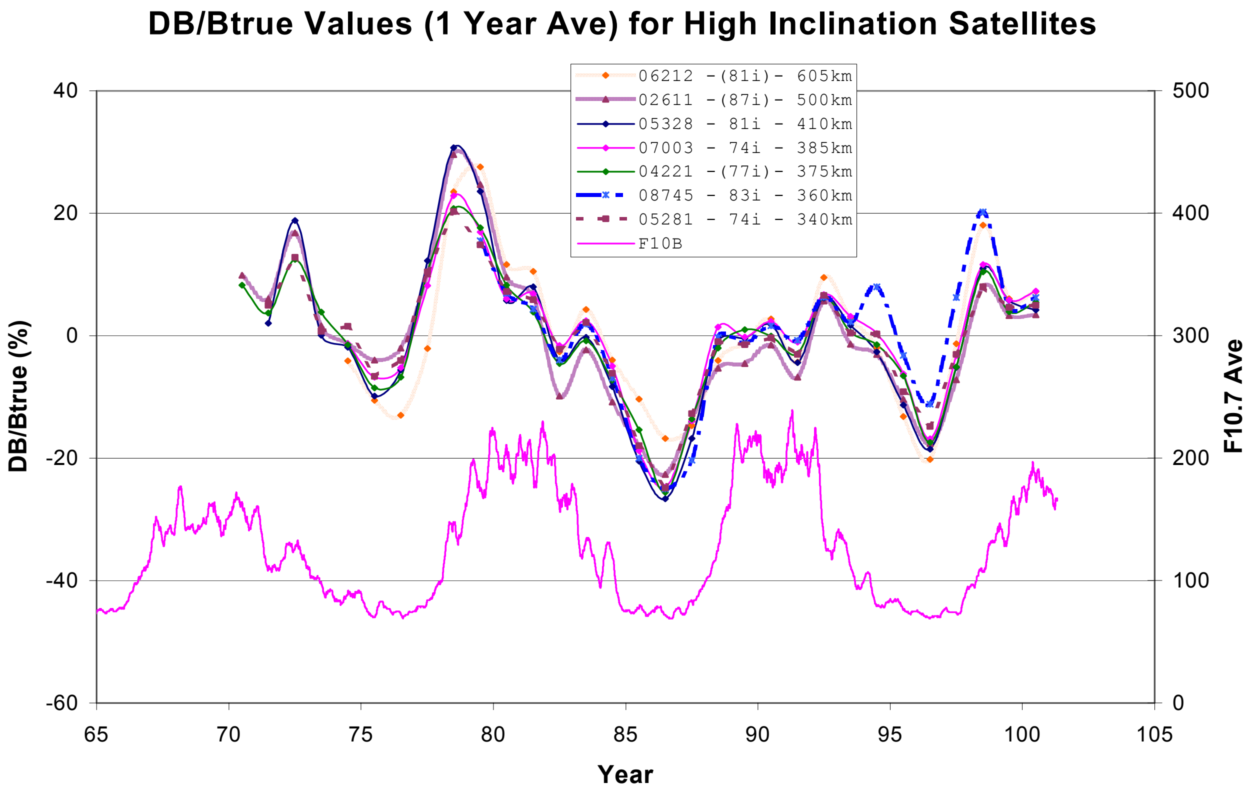

Figure 4. One year average B′ variations for satellites of high inclinations greater than 74 degrees.

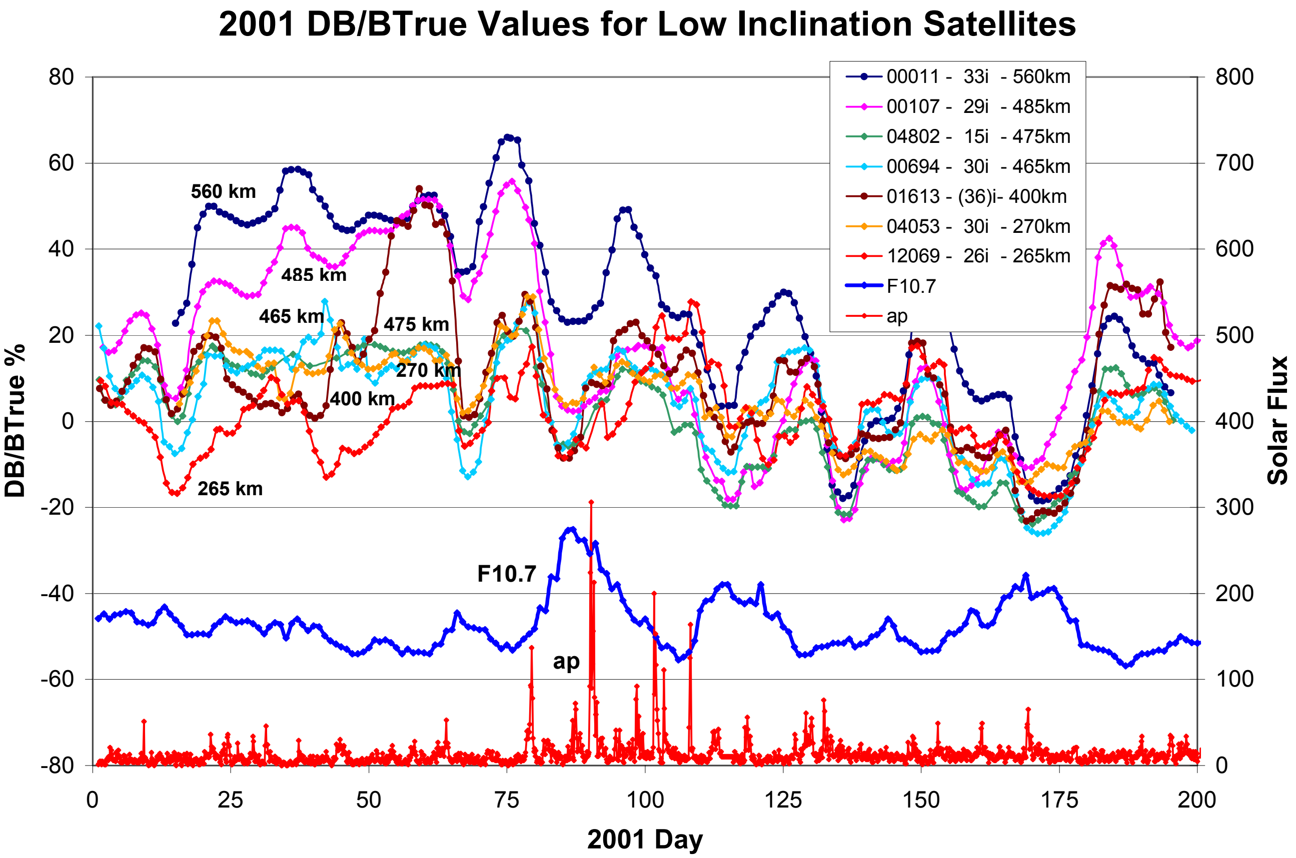

Figure 5. 2001 variations of B′ for low inclination satellites with perigee heights of 265 km to 560 km.