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HASDM Validation Tool Using Energy Dissipation Rates

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Abstract

An independent method for estimating dynamically varying upper atmospheric neutral density is used to validate the Air Force Space Battlelab's High Accuracy Satellite Drag Model (HASDM). This validation tool estimates neutral density near real-time using orbital energy dissipation rates (EDRs) from 75 trajectories of inactive payloads and debris, known as calibration satellites. EDRs are computed through a separate orbit determination algorithm that uses "segmented" estimates for the ballistic coefficient. This tool estimates density in terms of time-varying spherical harmonic expansions of the exospheric temperature and inflection point temperature, the key parameters characterizing the local density profile in the Jacchia models. In addition to validating HASDM density estimates, this tool is also used to optimize the density correction parameters and the set of calibration satellites. It could be implemented in operations as an alternate method for computing near real-time density corrections for improved trajectory predictions of high-drag satellites. It could also serve as a tool for developing and testing new upper atmospheric density and wind models.

Introduction

Atmospheric drag is the predominant error source in force models used to estimate and predict low-perigee satellite trajectories. Errors in current upper atmospheric density models of ±15% to 20% cause significant errors in predicted satellite positions [11]. Current empirical models typically model only 'climatological' variations in density as a function of season, time of day, position, and the classic solar/geomagnetic indices: $F_{10.7}, \bar{F}_{10.7}$ and a_p. In HASDM, 'meteorological' variations in density are estimated near real-time for the atmosphere above 90 km, the layer known as the thermosphere.

The High Accuracy Drag Model (HASDM) [18] is a Space Battlelab initiative begun in January 2001. The major objectives of HASDM are to estimate and predict neutral atmospheric density from observed satellite drag effects on low-perigee inactive payloads and debris, known as *calibration satellites*. In contrast to the EDR Validation Tool described here, HASDM processes satellite tracking data directly to estimate the density correction parameters, through the Dynamic Calibration Atmosphere (DCA) algorithm [5]. DCA and the EDR Validation Tool both use the Thermospheric Correction Model, a modified Jacchia 1970 (J70) model [9] employing spatially varying corrections as a function of latitude ϕ, local solar time λ, and altitude h. The coefficients of these corrections are allowed to change dynamically with time.

Research has shown it is important to capture the dynamic variability of diurnal and higher order density variations [6,8]. This is done here using a spherical harmonic expansion (in latitude ϕ and local solar time λ) of two independent corrections to the Jacchia temperature profile at all geographic locations. One of these corrections is applied to the nighttime minimum exospheric temperature T_c, the key parameter describing the state of the thermosphere in response to solar extreme ultraviolet (EUV) heating. The nighttime minimum exospheric temperature is the global minimum of the exospheric temperature T_∞ (temperature above ~600 km altitude). The other correction is applied to the local inflection point temperature T_x (temperature at 125 km altitude). Correcting T_c and T_x, using two independent spherical harmonic expansions, provides sufficient degrees of freedom to estimate local solar time λ variations whose phase and amplitude change dynamically with altitude h and latitude ϕ. Each local temperature profile leads to a co-located density profile through integration of the hydrostatic and diffusion equations. The spherical harmonic corrections, when applied to the standard J70 model's temperature parameters, lead to a corrected global density field as a function of ϕ, λ and h.

The inputs to the Thermospheric Correction Model are date, time, position, solar/geomagnetic indices and spherical harmonic coefficients for both the ΔT_c and the ΔT_x corrections. The outputs are total neutral density, spatial density gradients, and partial derivatives relating changes in density to changes in each spherical harmonic coefficient. The spatial density gradients in the upward, eastward and northward directions are used by some satellite orbit propagators, including DCA. The density partial derivatives with respect to the spherical harmonic coefficients are necessary for estimating these coefficients.

The EDR Validation Tool processes orbital energy dissipation rate (EDR) data from a collection of calibration satellites to produce estimates for the spherical harmonic correction coefficients. Calibration satellite orbits with different altitudes, orbital planes, eccentricities and perigee positions are exploited simultaneously to determine a time-varying global density field. To minimize orbit determination errors, these orbits are estimated using a full special perturbations orbit theory, including geopotential fluctuations, luni-solar (third body) gravitation, solar radiation pressure and, of course, atmospheric drag.

Since EDR is a quantity integrated along the orbit, it represents the cumulative effect of drag on the satellite trajectory over the estimation time span. In previous work, [17] this process was tested in simulation mode to verify that ΔT_c coefficients can be successfully recovered from the EDR values. The thermospheric correction model was further tested by recovering the true density as simulated by the MSISE-90 model [7,18]. In addition to validating DCA, the EDR Validation Tool was used to help optimize the density correction parameter set and the set of calibration satellites used in HASDM. It can also be used as tool to develop and test new thermospheric density and wind models. If implemented for satellite orbit determination, the resulting density field, estimated using the EDR Validation Tool or DCA, should significantly improve the accuracy of estimated and predicted trajectories for all low-perigee satellites.

Thermospheric Correction Model

This model is a modification of the standard Jacchia 1970 thermospheric density model [9]. The density field produced by the standard Jacchia 1970 (J70) model is determined by the date/time of interest and the solar/geomagnetic indices at this time. In HASDM, the new $E_{10.7}$ index is used in addition to $F_{10.7}$. $E_{10.7}$ is based on the solar extreme ultraviolet (EUV) radiation instead of the 10.7-cm solar radio flux. Therefore, $E_{10.7}$ represents the true EUV heating of the thermosphere. $E_{10.7}$ is scaled to the same range as $F_{10.7}$ and is designed as a substitute whenever a model calls for $F_{10.7}$. Joule heating, the other major heat source, is quantified by a_p, the planetary geomagnetic index.

An increase in heating leads to an expanded thermosphere with greater atmospheric densities above ~100 km altitude, the isopycnic (constant density) level. However, the traditional solar/geomagnetic indices do not accurately represent the true energy inputs to the thermosphere [10,13]. This adds to the inherent inaccuracies of the models themselves. The direct use of space surveillance observations, or the use of orbital EDR values deduced from calibration satellite trajectories, is superior to the sole use of imperfect solar/geomagnetic indices, since EDR values and satellite tracking observations reflect the real drag effects.

As represented in the J70 model, the exospheric temperature for a particular day of year and set of solar/geomagnetic indices is an empirical function of geocentric latitude ϕ and local solar time λ, and may be expressed as $T_\infty(\phi, \lambda)$. The standard Jacchia 1970 model produces a temperature profile at every location on Earth based on the exospheric temperature at that location. In the Thermospheric Correction Model, both the nighttime minimum exospheric temperature T_c and the inflection point temperature T_x are altered by spherical harmonic corrections.

Using the local temperature profile, the hydrostatic and diffusion equations are integrated, subject to the J70 lower thermosphere boundary conditions at 90 km altitude, to produce a corresponding neutral density profile. Above ~300 km, changing T_x has an effect similar to multiplying the model density by a constant factor, or equivalently, shifting $\log_{10}(\rho)$ by a constant without significantly affecting the *scale height*. This is the height interval over which the density changes by a factor of *e*. On the other hand, changing T_∞ alters the scale height as well as the density at all altitudes. Modifying the standard J70 density profile through these local temperature parameters ensures physical reality in the resulting density profile, since the hydrostatic and diffusion equations are satisfied.

The nighttime minimum exospheric temperature T_c is the global minimum in exospheric temperature, before the effects of geomagnetic activity and semiannual variation are applied. In the J70 model, T_c is computed empirically from the solar 10.7-cm radio flux $F_{10.7}$, and its centered mean over three solar rotations $\bar{F}_{10.7}$ (or equivalent $E_{10.7}$ indices). The spherical harmonic temperature corrections, $\Delta T_c(\phi,\lambda)$ and $\Delta T_x(\phi,\lambda)$, are expressed as the estimated temperature minus the corresponding standard Jacchia 1970 model temperature. The expression for the local exospheric temperature T_∞ is given by:

$$T_\infty(\phi,\lambda) = [T_c + \Delta T_c(\phi,\lambda)]\; D(\delta_*,\phi,\lambda) + \Delta T_G(a_p) + \Delta T_S(t,\overline{F}_{10.7}) \quad (1)$$

where $T_c \equiv$ Nighttime minimum exospheric temperature from standard J70 model (°K)
$D \equiv$ J70 diurnal variation function (dimensionless)
$\delta_* \equiv$ declination of the sun (rad)
$\phi \equiv$ geocentric latitude at point of interest (rad)
$\lambda \equiv$ local solar time at point of interest (rad)
$\Delta T_G \equiv$ J70 geomagnetic activity adjustment to T_∞(°K)
$a_p \equiv$ 3-hourly planetary geomagnetic index
$\Delta T_S \equiv$ J70 semiannual variation adjustment to T_∞ (°K)
$t \equiv$ Modified Julian Date (days)
$\overline{F}_{10.7} \equiv$ mean solar 10.7-cm radio flux (10^{-22} W m^{-2} Hz^{-1})

The following equation shows how $T_\infty(\phi,\lambda)$ and $\Delta T_x(\phi,\lambda)$ are applied to obtain the corrected $T_x(\phi,\lambda)$.

$$T_x(\phi,\lambda) = 444.3807 + 0.02385\, T_\infty(\phi,\lambda) - 392.8292\,\exp(-0.0021357\, T_\infty(\phi,\lambda)) + \Delta T_x(\phi,\lambda) \quad (2)$$

The spherical harmonic expansion of ΔT_c may be expressed as:

$$\Delta T_c(\phi,\lambda) = \hat{C}_{00} + \sum_{n=1}^{N}\left[\sum_{m=0}^{n}\hat{C}_{nm}P_{nm}(z)\cos(m\lambda) + \sum_{m=1}^{n}\hat{S}_{nm}P_{nm}(z)\sin(m\lambda)\right] \quad (3)$$

where $z \equiv \sin\phi$. The spherical harmonic expansion of ΔT_x may be similarly expressed as:

$$\Delta T_x(\phi,\lambda) = \tilde{C}_{00} + \sum_{n=1}^{N}\left[\sum_{m=0}^{n}\tilde{C}_{nm}P_{nm}(z)\cos(m\lambda) + \sum_{m=1}^{n}\tilde{S}_{nm}P_{nm}(z)\sin(m\lambda)\right] \quad (4)$$

The 'hat' symbol ^ identifies coefficients for the ΔT_c expansion and the 'tilde' symbol ~ identifies coefficients for the ΔT_x expansion. Here, n is the degree and m is the order of the spherical harmonic term. The functions P_{n0} are the normalized Legendre functions (polynomials) and the functions P_{nm}, $m > 0$, are the normalized associated Legendre functions [6]. There are a total of $(N+1)^2$ terms in a particular spherical harmonic series truncated to degree N.

The temperature profile is a function of T_x, T_∞ and altitude h and is given in the description of the Jacchia 1970 model [9]. The value for the total neutral density ρ, and its partial derivatives with respect to the local temperature parameters, $\partial\rho/\partial T_x$ and $\partial\rho/\partial T_\infty$, as well as its partial derivative with respect to altitude $\partial\rho/\partial h$, are interpolated from density tables. The tabular range for T_x is 200°K to 968.15°K, subject to the constraint that T_x be less than or equal to T_∞. The tabular range for T_∞ is 204.95°K to 3000°K. The tabular range for altitude h is from 0 km (surface) to 5000 km. The tabular ranges in temperatures T_x and T_∞ extend far beyond what is physically realistic. This is done to ensure that initial iterations of the differential correction estimation process do not wander off the edge of the density tables. From the surface to 125 km altitude, the standard MSISE-90 is also called for the total neutral density. Between 90 km and 125 km, the density from the Thermospheric Correction Model is blended with the standard MSISE-90 density [7] using cubic spline weighting as a function of altitude. Above 1500 km altitude, the standard Jacchia 1970 is used, but is multiplied by an exospheric density correction factor [2] given by the following equation:

$$f = 0.22 - 0.00210\,\overline{F}_{107} + 0.00116\,h + 0.00000213\,\overline{F}_{107}\,h \quad (5)$$

Between 1000 km and 1500 km, both this modified J70 exospheric model and the Thermospheric Correction Model are blended together using cubic spline weighting. Therefore, the spherical harmonic coefficients influence the density from 90 km to 1500 km altitude only.

EDR Validation Tool

This program is designed to estimate density correction parameters from calibration satellite trajectory information obtained from the orbit determination process. The "observations" input to this program are the orbital specific energy dissipation rates (EDRs) deduced from the estimated satellite trajectories. These EDRs are computed from the modeled ballistic coefficients estimated during orbit determination, the model density used for orbit determination, and the estimated satellite trajectories. The modeled ballistic coefficient B' is a biased estimate of the true ballistic coefficient B. Its value depends on the true ballistic coefficient B and biases in the standard (uncorrected) thermospheric model's neutral density [11,16]. The Thermospheric Correction Model described in the previous section was developed from the standard Jacchia 1970 (J70) model. The major modifications of the Thermospheric Correction Model with respect to the standard J70 model are the addition of input coefficients for the spherical harmonic expansions of ΔT_c and ΔT_x , and the addition of output partial derivatives of the total neutral density with respect to these coefficients. Using these density partial derivatives, one can evaluate the partial derivatives expressing the change in the dissipation rate of orbital specific energy $\dot{\varepsilon}$ with respect to changes in each spherical harmonic coefficient.

To estimate the spherical harmonic coefficients of the density correction from orbital energy dissipation rates (EDRs), the Thermospheric Correction Model must be called by a least squares iterative process. Through this iteration, successive values for the coefficients $\hat{C}_{nm}$, $\hat{S}_{nm}$, $\tilde{C}_{nm}$ and $\tilde{S}_{nm}$ are estimated for each time span. The spherical harmonic coefficients are all initialized to zero before entering the iteration loop. This is equivalent to initializing the density field to that of the standard J70 model. Within the least squares iteration, the residuals between the "observed" and "computed" specific energy dissipation rates are computed for all satellites. The residual for the i^{th} satellite is $\delta\dot{\varepsilon}_i = \dot{\varepsilon}_i' - \dot{\varepsilon}_i$ The sum of the squares of these residuals is minimized in the least squares iteration. The specific energy dissipation rates are given by the following expressions:

$$\dot{\varepsilon}_i = \frac{B_i}{2\Delta t}\int_0^{\Delta t} \rho_i \; V_{\text{rel}}\,(\mathbf{V}_{\text{rel}}\cdot\mathbf{V}_{\text{sat}})\,dt \qquad \text{computed by EDR Validation Tool 'computed'} \qquad (6)$$

$$\dot{\varepsilon}_i' = \frac{B_i'}{2\Delta t}\int_0^{\Delta t} \rho_i' \; V_{\text{rel}}\,(\mathbf{V}_{\text{rel}}\cdot\mathbf{V}_{\text{sat}})\,dt \qquad \text{modeled by orbit determination process 'observed'} \qquad (7)$$

where B_i ≡ 'true' ballistic coefficient for i^{th} satellite ($m^2\ kg^{-1}$)
B_i' ≡ modeled ballistic coefficient for i^{th} satellite ($m^2\ kg^{-1}$)
Δt ≡ length of fit span time interval (s)
ρ_i ≡ 'true' density along satellite trajectory ($kg\ m^{-3}$)
ρ_i' ≡ model density along satellite trajectory ($kg\ m^{-3}$)
$\mathbf{V}_{\text{rel}}$ ≡ velocity of satellite relative to the atmosphere ($m\ s^{-1}$)
V_{rel} ≡ magnitude of $\mathbf{V}_{\text{rel}}$ ($m\ s^{-1}$)
$\mathbf{V}_{\text{sat}}$ ≡ velocity of satellite in inertial frame ($m\ s^{-1}$)
dt ≡ differential element of time (s)

The 'computed' value is determined using the best estimate for the true B_i (long-term average) and 'true' ρ_i (computed by the Thermospheric Correction Model). The 'observed' value is determined from the modeled density ρ_i' used in the orbit determination process and the associated B_i' estimated during orbit determination. Other important elements of the least squares process are the partial derivatives relating a change in the 'observable' (EDR) to a corresponding change in one of the estimated density coefficients. The 'true' density ρ_i depends on the estimated spherical harmonic coefficients for ΔT_c (ϕ,λ) and $\Delta T_x(\phi,\lambda)$. The partial derivatives of $\dot{\varepsilon}_i$ with respect to the spherical harmonic coefficients for ΔT_c and ΔT_x respectively are:

$$\frac{\partial\dot{\varepsilon}_i}{\partial\hat{C}_{nm}} = \frac{B_i}{2\,\Delta t}\int_0^{\Delta t}\frac{\partial\rho_i}{\partial\hat{C}_{nm}}\;V_{\text{rel}}\,(\mathbf{V}_{\text{rel}}\cdot\mathbf{V}_{\text{sat}})\,dt \quad \text{and} \quad \frac{\partial\dot{\varepsilon}_i}{\partial\hat{S}_{nm}} = \frac{B_i}{2\,\Delta t}\int_0^{\Delta t}\frac{\partial\rho_i}{\partial\hat{S}_{nm}}\;V_{\text{rel}}\,(\mathbf{V}_{\text{rel}}\cdot\mathbf{V}_{\text{sat}})\,dt \qquad (8)$$

$$\frac{\partial\dot{\varepsilon}_i}{\partial\tilde{C}_{nm}} = \frac{B_i}{2\,\Delta t}\int_0^{\Delta t}\frac{\partial\rho_i}{\partial\tilde{C}_{nm}}\;V_{\text{rel}}\,(\mathbf{V}_{\text{rel}}\cdot\mathbf{V}_{\text{sat}})\,dt \quad \text{and} \quad \frac{\partial\dot{\varepsilon}_i}{\partial\tilde{S}_{nm}} = \frac{B_i}{2\,\Delta t}\int_0^{\Delta t}\frac{\partial\rho_i}{\partial\tilde{S}_{nm}}\;V_{\text{rel}}\,(\mathbf{V}_{\text{rel}}\cdot\mathbf{V}_{\text{sat}})\,dt \qquad (9)$$

$\partial\rho_i/\partial\hat{C}_{nm}$, $\partial\rho_i/\partial\hat{S}_{nm}$, $\partial\rho_i/\partial\tilde{C}_{nm}$ and $\partial\rho_i/\partial\tilde{S}_{nm}$ are provided by the Thermospheric Correction Model. The 'true' ballistic coefficient B_i is estimated by computing the exponential mean (backward weighted average) of the ballistic coefficients, estimated using the corrected density. The *e*-folding for this mean is typically set to 1 day. However, this exponential mean is constrained using an *a priori* long-term (many years) average of the estimated ballistic coefficient B_i', whenever such an average is available.

As the residual for each of the satellites is computed, the associated partial derivatives relating changes in energy dissipation rate to changes in the spherical harmonic coefficients are also computed. These partial derivatives are the elements of the parameter sensitivity matrix **H**. This matrix has M rows and L columns, where M is the total number of calibration satellites used in the time span and L is the total number of spherical harmonic coefficients estimated.

If each energy dissipation rate residual is assumed to be linearly independent of the others, then the observation weighting matrix **W** is diagonal. The weight w_i for each satellite may be determined empirically as the inverse of the variance of the EDR residuals over many fit spans [1]. The **W** matrix has M rows and M columns. With the expressions for the parameter sensitivity matrix **H**, the weighting matrix **W**, and the residual vector $\delta\dot{\boldsymbol{\varepsilon}}$ defined, one can compute the differential correction $\delta\mathbf{C}_{nm}$ to the coefficients using the normal equation:

$$\delta\mathbf{C}_{nm} = (\mathbf{H}^{\mathrm{T}}\mathbf{W}\ \mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{W}\,\delta\dot{\boldsymbol{\varepsilon}} \qquad (10)$$

This differential correction vector is used to correct the spherical harmonic coefficients vector from the previous iteration. The least squares iteration converges when the root mean square (rms) of the energy dissipation rate residuals $\delta\dot{\varepsilon}_i$ no longer changes significantly from one iteration to the next. A separate set of spherical harmonic coefficients is estimated for each time span.

Interpolation, Extrapolation and Filtering for the Estimation Process

The primary inputs to the EDR Validation Tool are the ephemerides of the calibration satellites, the segmented ballistic coefficients, and the solar/geomagnetic indices. The ephemerides consist of positions and velocities of the calibration satellites at 1-minute intervals. The segmented ballistic coefficients are biased ballistic coefficients estimated over half-hourly or hourly intervals. They are biased because of errors in the standard Jacchia 1970 (J70) model. For algorithmic simplicity, the hourly values are represented as two adjacent and equal half-hourly values. The solar/geomagnetic indices consist of the 3-hourly planetary geomagnetic index a_p, the daily 10.7-cm solar radio flux $F_{10.7}$, and its centered mean $\bar{F}_{10.7}$ averaged over three solar rotations (81 days).

As a_p changes abruptly from one 3-hour interval to the next, and as $F_{10.7}$ and $\bar{F}_{10.7}$ change from one day to the next, the density provided by the standard J70 model also changes abruptly. Since the EDR Validation Tool estimates a smoothly varying spherical harmonic expansion, a sudden change in the standard 'base' density being corrected induces false contributions to the spherical harmonics that are artifacts of these step functions of density with time. Therefore, it is important to smooth out the solar/geomagnetic indices using an interpolation process. Since the finest time resolution in the segmented ballistic coefficient is a half-hour, the solar/geomagnetic indices are interpolated to half-hour values. This interpolation is done using quadratic interpolation polynomials subject to the constraint that the average value of the polynomial integrated over the original index interval (3 hours for a_p or 1 day for $F_{10.7}$ and $\bar{F}_{10.7}$) is equal to the original value of the index over that interval. Adjacent overlapping quadratic polynomials are blended together using quadratic B-splines [15]. This interpolation process results in relatively continuous changes in the standard model density as a function of time, thus reducing the noise in the estimated spherical harmonic coefficients.

It was discovered that 1-minute calibration satellite ephemerides have insufficient temporal resolution to compute accurate energy dissipation rates, especially for satellites with relatively high eccentricity ($e > 0.1$). Therefore, the ephemerides were interpolated to 5-second intervals. This was done using Hermite polynomial interpolation.

While solving for the state vectors of the calibration satellites over a 1.5 to 3-day fit span, the segmented approach simultaneously estimates a different ballistic coefficient for hourly or half-hourly segments. Allowing the estimated ballistic coefficient to vary throughout the fit span provides EDRs at a much finer time resolution than one fit span. The 5-second satellite ephemerides, together with the half-hourly segmented ballistic coefficients, and the standard J70 density are used to integrate the 'observed' energy dissipation rates (EDRs). The 'modeled' EDRs are computed from the same 5-second ephemerides, *a priori* 'true' ballistic coefficient, and the Thermospheric Correction Model density, corrected through the estimated spherical harmonic temperature coefficients. While computing the model EDRs, the partial derivatives of EDR with respect to the spherical harmonic coefficients are integrated in the same manner. A third-order Overhauser interpolation scheme [14] is used to integrate the EDRs and their derivatives over each half-hour segment.

It was soon discovered that the resulting half-hour EDRs (both 'observed' and 'modeled') are too noisy for use in estimating spherical harmonic coefficients directly. This is primarily due to errors in the segmented ballistic coefficients, presumably due to errors in the space surveillance tracking observations. Therefore, it was decided to average the EDRs and associated partial derivatives over several contiguous segments. Rather than compute a simple arithmetic mean over a series of contiguous segments, a weighted mean was found to produce smoother, less noisy results. The weighting scheme employs a quadratic B-spline, which looks somewhat like a normal Gaussian distribution. Like the normal distribution, the quadratic B-spline is bell-shaped and integrates to unity. However, unlike the normal distribution, the quadratic B-spline has a finite domain, making it better suited for efficient computation, since it does not involve an infinite number of contiguous segmented values. The time interval spanned by the B-spline is referred to as a *section*. If the B-spline extends over an 18-hours interval, the middle 6 hours contains 2/3 of the weight and each of the 6-hour wings contains 1/6 of the weight. Such a section is made up of 36 contiguous half-hour segments. The EDR Validation Tool advances each section one half-hour at a time. This averaging technique produces results similar to a simple 6-hour arithmetic averaging scheme, but is much smoother. This approach ultimately produces time series of spherical harmonic coefficients having good temporal resolution while exhibiting a continuous behavior with respect to time.

The EDR Validation Tool requires *a priori* estimates for each of the true ballistic coefficients B_i. These values are obtained through a backward exponentially weighted mean of the 'corrected' ballistic coefficients. The *e*-folding of the exponential weighting is typically set to 1 day. Corrected ballistic coefficients are the ballistic coefficients that result after the spherical harmonic density correction is applied. These estimates for the true ballistic coefficient are initialized to either a 30-year average [3] of the estimated ballistic coefficient using the standard Jacchia 1970 model or an average of the estimated ballistic coefficient over the 180-day HASDM evaluation period. No matter what value is used to initialize the *a priori* ballistic coefficient, the exponential mean of the corrected ballistic coefficient eventually (after about 6 days) settles into an unambiguous time series of *a priori* values. The 'true' ballistic coefficients computed through this exponential mean are subject to an *a priori* constraint, so as to deviate from the initial value with a standard deviation of 3%. These exponentially averaged, constrained ballistic coefficients are then used as *a priori* values for the 'true' ballistic coefficients in the density estimation iteration for the next section of data.

The same backward exponentially weighted mean was applied to compute the variance in the EDR residuals between observed and modeled EDR. This variance is used to de-weight satellites having a high EDR variance. For each calibration satellite, an exponentially averaged EDR and associated EDR variance is computed. The resulting values are plotted for each satellite as a function of exponentially averaged EDR. A simple variance model is then fit to the plotted values. This model is of the form:

$$\sigma^2 = a_0 + a_1 \dot{\varepsilon}^2 \tag{11}$$

To apply the variance model, the EDR of the calibration satellite for the current section is plugged into the empirical variance formula to produce an empirical variance for the satellite. The inverse of this empirical variance is the weight w_i applied to each calibration satellite in the EDR Validation Tool.

Finally, a cubic extrapolation scheme is used to predict the best first guess of the spherical harmonic coefficients for the next section in the time series. Since the B-spline averaging smooths the temporal variation of the coefficients, this extrapolation technique is accurate in predicting the next values for the coefficients. Therefore, this extrapolation significantly accelerates the convergence of the differential correction iteration for the next set of coefficients. This technique typically reduces the number of iterations required by about half.

Comparison of Validation Tool to Dynamic Calibration Atmosphere

The time series of the spherical harmonic coefficients produced by the EDR Validation Tool are very similar to those produced by the Dynamic Calibration Atmosphere (DCA). The DCA spherical harmonic temperature/density coefficients may be slightly more accurate, due to direct processing of the space surveillance observations and the fact that the orbital trajectories are adjusted while the density coefficients are being estimated. However, the coefficient time series from the EDR Validation Tool appears to contain more detailed information (at least qualitatively) since its B-spline average is advanced

1/2 hour at a time, instead of every 3 to 12 hours, as is done in DCA. The following figures compare temperature/density correction coefficients estimated by DCA with those estimated by the EDR Validation Tool. In these figures, the choppy green line represents the DCA-generated coefficients and the smooth pink line represents coefficients generated by the EDR Validation Tool. The red line at the bottom represents the 3-hourly planetary geomagnetic a_p index and the blue line just above it represents the $F_{10.7}$ solar radio flux. The coefficients generated by the EDR Validation Tool vary more smoothly due to the quadratic B-spline averaging and the fact that this averaging filter is advanced one half hour at a time. The results for the zeroth-degree coefficients and the first-degree coefficients for ΔT_x were discussed in another paper at this Conference [4]. Figure 1 displays an example of a normalized spherical harmonic as a function of latitude and local solar time; in particular, the function corresponding to the $\hat{C}_{21}$ coefficient, when the coefficient is set to 1. Figures 2 through 9 display the degree 1 and degree 2 coefficients for ΔT_c. Each of these figures has an insert that displays the spherical harmonic function in the same manner as Figure 1.

The following is a description of the time series (Figures 2, 3 and 4) for each of the first-degree spherical harmonic coefficients for the nighttime minimum exospheric temperature correction ΔT_c.

- $\hat{C}_{10}$ is moderately anti-correlated with the planetary geomagnetic index a_p.
- $\hat{C}_{11}$ exhibits a strong positive correlation with enhanced a_p.
- $\hat{S}_{11}$ also exhibits a strong positive correlation with enhanced a_p.

The following is a description of the time series (Figures 5 through 9) for each of the second-degree spherical harmonic coefficients for the nighttime minimum exospheric temperature correction ΔT_c:

- $\hat{C}_{20}$ appears to have a weak positive correlation with a_p in the initial phase of the geomagnetic storms, followed by a weak anti-correlation.
- $\hat{C}_{21}$ reacts either positively or negatively to geomagnetic storms depending on the storm. For example, the storm on day 90 exhibits a strong positive reaction where as the storm on day 101 exhibits a strong negative reaction.
- $\hat{S}_{21}$ appears to have a weak positive correlation with the initial phase of geomagnetic storms, followed by a weak anti-correlation.
- $\hat{C}_{22}$ exhibits a moderately positive correlation with geomagnetic storms.
- $\hat{S}_{22}$ exhibits a very weak positive correlation with the initial phases of the geomagnetic storms, followed by a weak negative reaction.

Conclusion

The Energy Dissipation Rate (EDR) Validation Tool is designed for use in estimating the neutral density field for the Space Battlelab's High Accuracy Satellite Drag Model (HASDM) initiative. This Validation Tool uses orbital parameters from a set of low-perigee satellites over a series of time spans as input. Using EDR values deduced from the orbit determination process, this program estimates the global neutral density field for each time span. Parameters are estimated describing the density field through the use of the Thermospheric Correction Model, a modified version of the Jacchia 1970 density model. The estimated density parameters for each time span are coefficients of two spherical harmonic expansions. One is an expansion of the nighttime minimum exospheric temperature difference ΔT_c; the other is an expansion of the local inflection point temperature difference ΔT_x. These temperature correction expansions lead to a corrected density field producing the "best fit" to the true density field. The estimated spherical harmonic coefficients for ΔT_c and ΔT_x specify a unique temperature profile at every latitude and local solar time and ultimately a unique value for the atmospheric density at every point in the thermosphere. The model converts temperature profiles into density profiles through the integration of the hydrostatic and diffusion equations subject to fixed lower boundary conditions at 90 km altitude. This integration is done once to generate density tables from which density and its derivatives are interpolated.

An important feature of the EDR Validation Tool is its ability to make use of energy dissipation rates from a variety of low-perigee satellites. Orbits with different altitudes, inclinations, and eccentricities are processed simultaneously. The more complete the global coverage of the calibration satellites, the more accurate are the estimated density parameters. The number of parameters can be expanded to include higher degree spherical harmonic truncations, if the parameter error covariance matrix shows small errors relative to the magnitude of the highest degree coefficients. The spatially varying corrections described here should produce a significantly more accurate density solution. The estimated spherical harmonic coefficients can be readily used to specify a corrected global density field which can be applied to improve the accuracy of special perturbations orbit determination and prediction for any low-perigee satellite. For the Space Battlelab's HASDM initiative, this validation tool was used to help optimize the calibration satellite set and the solution set, as well as to validate HASDM's Dynamic Calibration Atmosphere algorithm. The coefficient time series estimated by the Validation Tool agree well with those estimated by the Dynamic Calibration Atmosphere (DCA). The EDR Validation Tool may also be used to test the performance of new thermospheric density models.

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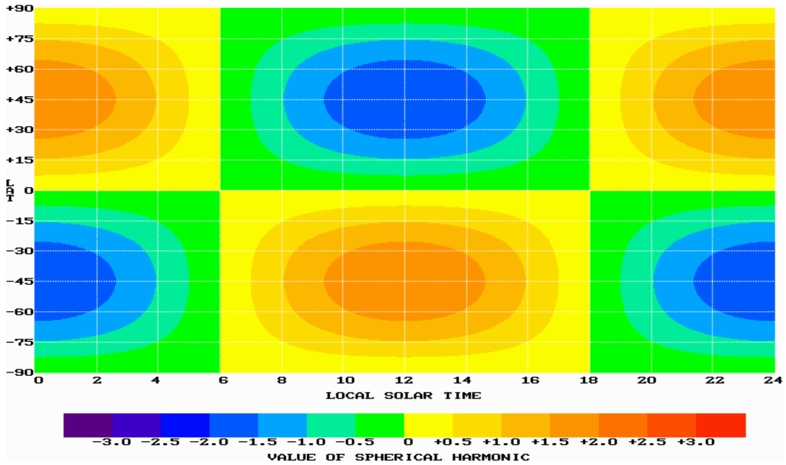

Figure 1. Example: Spherical Harmonic Function corresponding to $\hat{C}_{21} = 1$

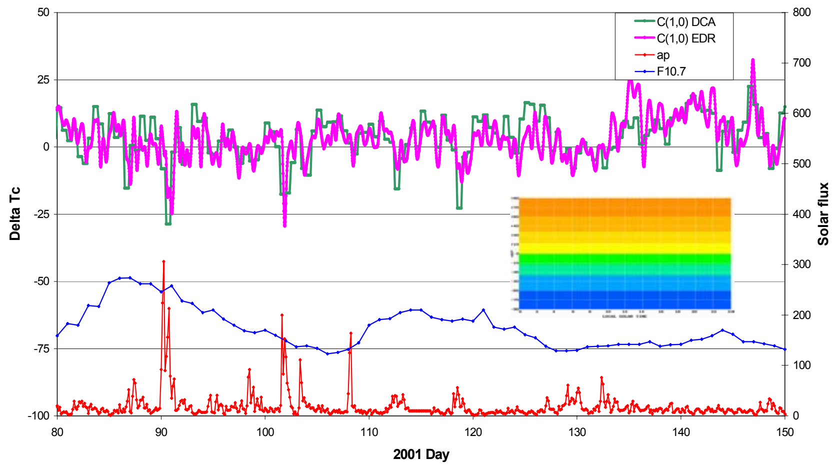

Figure 2. $\hat{C}_{10}$ from day 80 to 150

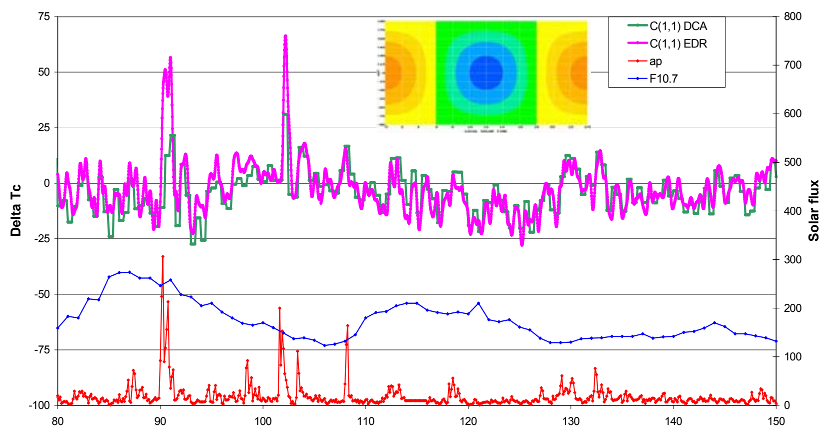

Figure 3. $\hat{C}_{11}$ from day 80 to 150

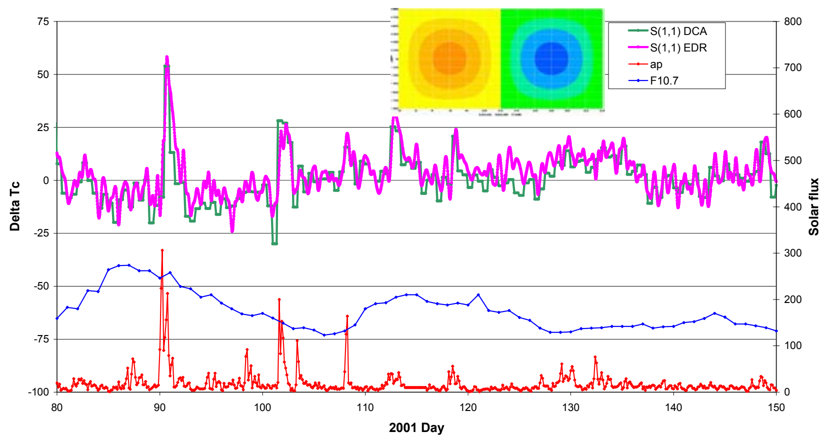

Figure 4. $\hat{S}_{11}$ from day 80 to 150

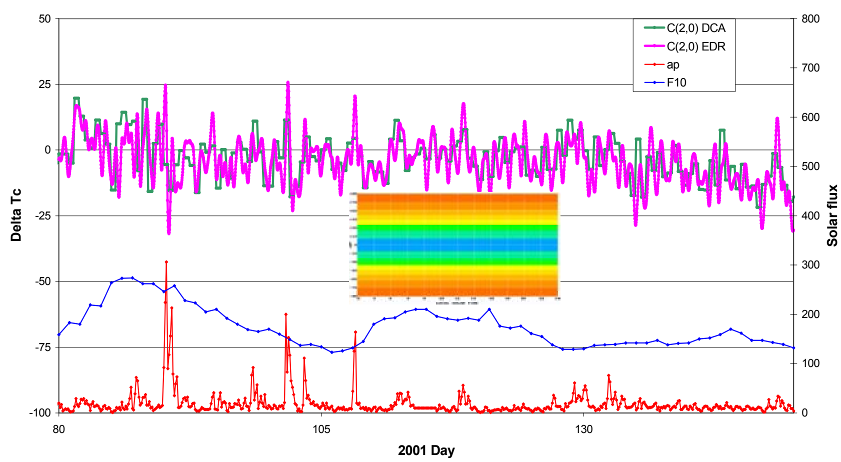

Figure 5. $\hat{C}_{20}$ from day 80 to 150

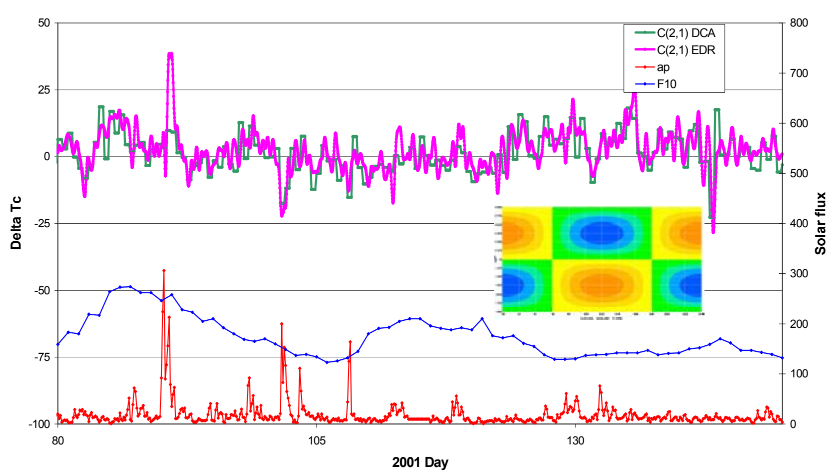

Figure 6. $\hat{C}_{21}$ from day 80 to 150

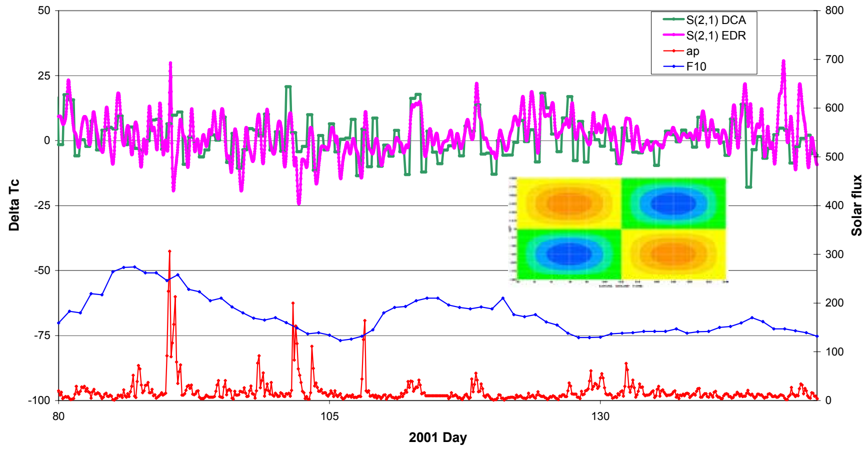

Figure 7. $\hat{S}_{21}$ from day 80 to 150

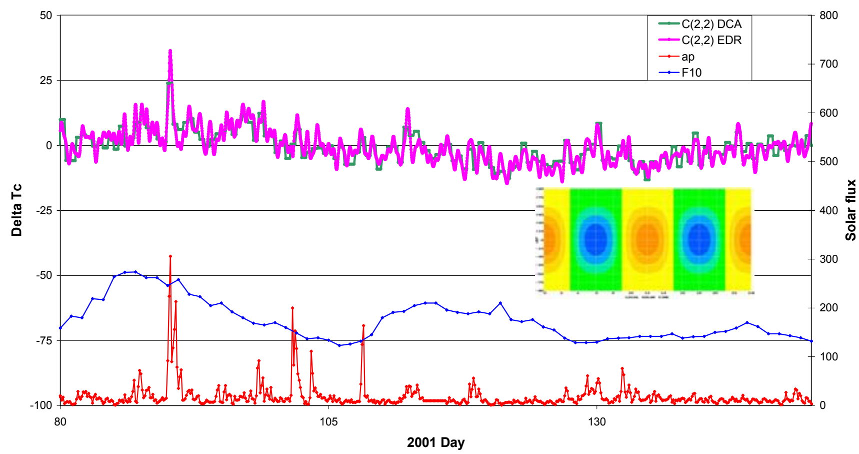

Figure 8. $\hat{C}_{22}$ from day 80 to 150

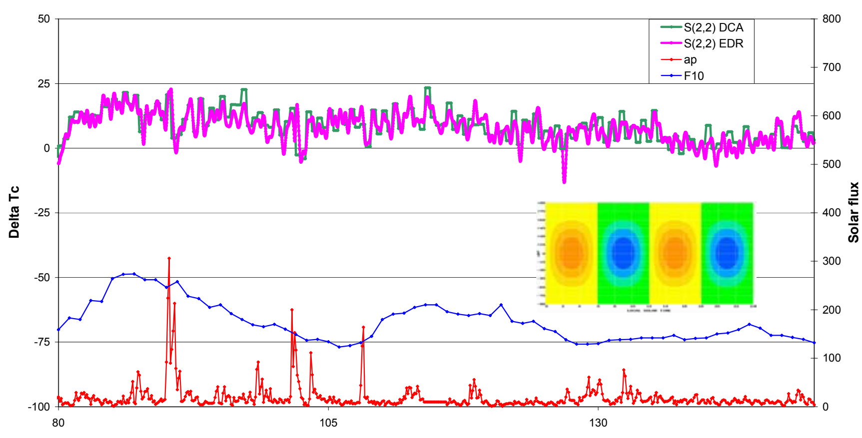

Figure 9. $\hat{S}_{22}$ from day 80 to 150