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N_2 POSITIVE AND N_2^+ BAND SYSTEMS AND THE ENERGY SPECTRA OF AURORAL ELECTRONS

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Abstract—The relative emission rates of the auroral N_2 positive and N_2^+ band systems can be used to limit the permissible range of differential electron fluxes in auroras, due to remarkable differences in electron excitation functions for the two kinds of systems. Use of recently measured electron cross sections and many observational data from ground based and rocket studies shows that the results are consistent with spectra equivalent to a power law $E^{-1.4}$ for primaries and secondaries combined. The unified primary spectra of Rees (1969) and secondary spectra of Rees *et al.* (1969) fails seriously to predict the optical ratios. It is shown that Rees' primary spectrum is deficient in slow primaries owing to the use of defective Monte Carlo results of Maeda (1965). Doubt is thereby cast on the validity of experimental results for the differential spectrum below 50 eV reported by Feldman *et al.* (1971) because of the rapid decrease in flux with energy shown by those measurements.

1. INTRODUCTION

Recently Feldman and his colleagues (Feldman *et al.*, 1971) have published the results of a measurement of the energy spectrum of slow electrons in an aurora. After correcting raw data from their electrostatic analyzer for a large background contribution, presumed to be produced by auroral ultraviolet radiation, they obtain a spectrum which varies as $E^{-3.17}$ in the range of electron energy E from 6 to 50 eV. This energy dependence holds over a span of altitudes from 100 to 155 km. Before it had been corrected the apparent flux varied as $E^{-1.35}$. The corrected spectrum agrees well, except for details in structure, with the energy dependence of *secondary electrons* in an aurora as calculated by Rees *et al.* (1969). If this agreement is accepted then it follows that, at least for the class of auroras under consideration, almost all electrons with energies up to 50 eV over the entire range of altitude in question are secondaries. This result would also tend to agree with a calculation by Rees (1969) who finds that below 150 km very few primaries with energy less than about 1 keV are to be found in the degraded primary flux virtually independent of the primary energy spectrum. Combining the spectra of primary electrons predicted by Rees (1969) with the spectra of secondaries calculated by Rees *et al.* (1969) for the same initial energy flux would yield a spectrum with a great chasm between about 0·1 and 1·0 keV. (See Fig. 21 of the review by Rees, 1969.) It should be noted that the calculations of secondary and primary spectra referred to here are independent and decoupled. The production rate of secondaries of a given energy is obtained either from the observed ionization rate (N_2^+ optical emission data) or from a predicted ionization rate based on the initial primary energy spectrum and an energy dissipation function. The flux of secondaries as a function of their energy is then computed from these production rates and the stopping power of the neutral and ionized atmosphere. The calculation of degraded primary spectra performed by Rees started from a Monte Carlo calculation of Maeda (1965) as expressed analytically by Maeda and Aiken (1968) that gave differential energy spectra of initially monoenergetic electrons at various penetration depths in an atmosphere. Rees has computed the effect of imposing a number of supposed initial energy spectra on these distributions.

Another recent calculation of the flux of residual primary electrons as a function of energy at various penetration depths is that due to Stolarski (1968). He uses experimentally determined energy dissipation rates for electrons as functions of penetration depth to obtain residual primary spectra from an assumed initial primary spectrum. He does not calculate the differential flux of secondary electrons but stops his calculation after presenting differential volume production rates. Nor does he attempt to show the primary spectra below 100 eV, owing to the practical experimental difficulty of distinguishing degraded primaries from secondaries at low energy. Some caution is needed in reading the literature, incidentally, because of an unfortunate tendency of some authors to refer to all slow auroral electrons as secondaries as defined for example, by Rees *et al.* (1969). Rocket borne electron energy analyzers cannot distinguish, of course, between primary and secondary electrons. In fact, partly at issue in this article is the question of the ratio of primaries to secondaries in the energy range around 100 eV.

It is the purpose of this article to examine to what extent this ensemble of information concerning electron energy distributions in auroras is consistent with certain optical excitation features of auroras. These optical properties appear to be characteristic of a wide class of auroras that have been studied—indeed all on which there are observations available. In particular, we compare the observed efficiencies for excitation of the positive band systems of N_2 with systems of N_2^+. The former are sensitive almost entirely to electrons in the spectrum below 40 eV while the latter can be excited effectively by electrons in a broad range of energies from 20 eV to the kilovolt range. We shall show that auroral optical data cannot be explained by a combination of the theoretical secondary electron spectra of Rees *et al.* (1969)—or the observed slow electron spectrum of Feldman *et al.* (1971)—and primary spectra of the sort predicted by Rees (1969). The difficulty seems to be that there is a decided deficiency in the flux of low energy degraded primaries (50 eV–1 keV) at all altitudes below 150 km in the theoretical spectra; the predicted N_2^+ band systems are far too weak compared to those observed. And in fact we show that the electrons in the population predicted by Rees (1969) are incapable of producing more than a small fraction of the ionization (or N_2^+ first negative band excitation) inherent in the incident primary flux. We suggest that there are many more primary electrons at all altitudes in the energy range from about 50 eV to 1 keV than Rees predicts using the Maeda (1965) results. The possibility that there is also a deficiency in the theoretical secondary spectrum in this energy range cannot be excluded either.

In view of the uncertainty with regard to the actual flux of slow, energy degraded, primary electrons to be expected at 50 eV it does not seem to be possible to make a meaningful comparison of the results of Feldman *et al.* with theory. As we have already remarked these results would admit of virtually no slow primaries below 50 eV if the theoretical secondary spectrum is correct. In fact we argue that the interpretation made by Feldman *et al.* (1971) would allow for only very small fluxes out to 800 eV. According to our analysis this is not in accord with the electron spectral characteristics required to explain what we regard as well established optical auroral characteristics.

The primary spectra obtained by Stolarski do not seem to have the same deficiency of slow degraded primaries that affect the Rees results. Unfortunately, since they begin at 100 eV, they cannot be compared with the experimental fluxes below 50 eV.

2. THEORY AND NOMENCLATURE

The volume emission rate of a band is written

$$I_{v'v''} = N_{v'}A_{v'v''} \tag{1}$$

where the symbols have their usual meaning. If there is no quenching we may also write

$$I_{v'v''} = g_{v'}A_{v'v''}/A_{v'} \tag{2}$$

where $g_{v'}$ is the population rate.

$$I = \sum_{v'}\sum_{v''} I_{v'v''} = \sum_{v'}\sum_{v''} g_{v'}A_{v'v''}/A_{v'} = g \tag{3}$$

is the total volume emission rate.

For excitation by electrons the population rate ($g_{v'}$) is given by

$$g_{v'} = [N_2X]\int Q_{v'}\Phi \, dE \tag{4}$$

and the excitation efficiency,

$$\Gamma_{v'} = g_{v'}/[N_2X], \tag{5}$$

where $[N_2X]$ is the ground state population density and $Q_{v'}$ is the excitation cross-section of level v'. The values of $g_{v'}$ as observed in the aurora along with physical considerations place a constraint on the possible shape functions of the differential electron flux (Φ).

3. RELATIVE EMISSION RATES OF N_2 AND N_2^+ SYSTEMS

The relative emission rates, which are almost certainly equal to the population rates at normal auroral altitudes, can be obtained from Equation (4) using measured excitation cross-sections, if one provides an electron distribution function.

The relative g_v values have been calculated using a distribution function $\Phi \alpha E^n$. We have performed this exercise because it provides us with an analytically simple device for characterizing the overall 'hardness' of the electron flux and not because we believe that the flux at all altitudes and energies can necessarily be described by a power law spectrum. This paper must not be regarded as an attempt to promote a universal power law spectrum for auroral electrons. The predicted relative emission rates are given in Fig. 1 as a function of the spectral index (n). It is clear that the emission rates of the N_2 positive systems relative to the N_2^+ systems are quite sensitive to the value of the spectral index. Note that the relative rates of the two positive systems and the two N_2^+ systems considered separately show very little variation. The $N_2$1PG emission rates used in the production of the graphs include the cascade contribution from the $N_2$2PG. The excitation functions used in the calculation are due to Shemansky and Broadfoot (1971a) for the N_2 positive systems at low energies, extended to higher energies through the measurements of Aarts *et al.* (1969) and Brinkmann and Trajmar (1970). The measured thresholds of the cross-sections according to Shemansky and Broadfoot correspond to the spectroscopic energies of the excited levels above the ground state. Table 1 gives the peak values of the cross-sections along with the corresponding energies. The values given in the Table for the N_2^+1N system are based on the measurements by Borst and Zipf (1970), and those for the N_2^+M system are due to Shemansky and Broadfoot (1971a). Transition probabilities used in the calculation were obtained from the tables given by Shemansky and Broadfoot (1971b).

A measure of the spectral index (n) corresponding to the auroral emission rates can be obtained by plotting the various observations on the appropriate curves given in Fig. 1. A number of independent auroral observations are available in the literature, and the average values have been plotted as circled points. All of the points fall in the $n = -1{\cdot}2$ to $-1{\cdot}6$ region.

A number of details concerning the auroral measurements should be kept in mind in the following discussion. The relative emission rates of the N_2 positive and N_2^+ systems in the aurora vary on a short time scale, generally over a factor of about 2. According to

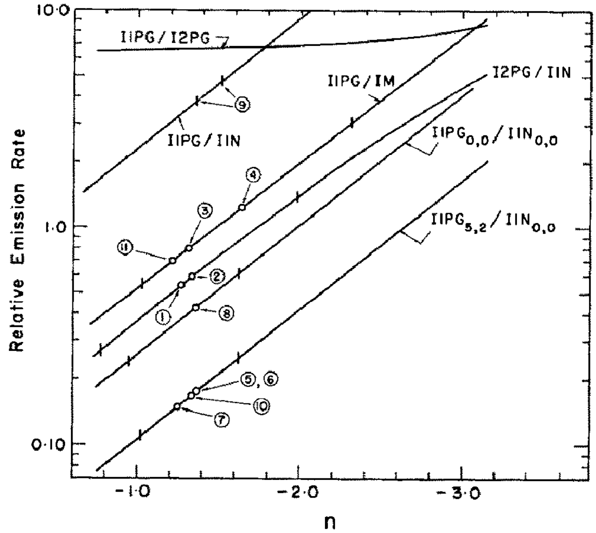

FIG. 1. RELATIVE EMISSION RATES OF N_2 AND N_2^+ SYSTEMS AS A FUNCTION OF ELECTRON ENERGY DISTRIBUTION.*

(1) Hunten (1955), average $IN_22PG_{0,2}/IN_2^+1N_{0,0}$.
(2) Petrie and Small (1952), average $IN_22PG/IN_2^+1N_{0,0}$.
(3) Chamberlain (1961), average IN_21PG/IN_2^+M.
(4) Shemansky and Vallance Jones (1968), average $IN_21PG_{4,1}/IN_2^+M_{3,0}$.
(5) NASA Flight 4.163, average $IN_21PG_{5,2}/IN_2^+1N_{0,0}$.
(6) NASA Flight 4.217, average $IN_21PG_{5,2}/IN_2^+1N_{0,0}$.
(7) NASA Flight 4.309, average $IN_21PG_{5,2}/IN_2^+1N_{0,0}$.
(8) Harrison (1969), average $IN_21PG_{0,0}/IN_2^+1N_{0,0}$.
(9) Recalculated from Stolarski (1968) theoretical work, $n = -1·36$ @ 110 km, $n = -1·52$ @ 170 km (see text).
(10) Gattinger and Vallance Jones (1971), average $IN_21PG_{5,2}/IN_2^+1N_{0,1}$.
(11) Gattinger and Vallance Jones (1971), average $IN_21PG_{5,2}/IN_2^+M_{4,1}$.

* Differential electron flux $\Phi \alpha E^n$.

TABLE 1. MEASURED PEAK ELECTRON CROSS SECTIONS (Q_v) OF N_2 AND N_2^+ STATES FOR EXCITATION FROM $N_2X^1\Sigma_g$

v	E_{TH}/E_p* (eV)	$C^3\Pi_u$† ($\times 10^{-17}$ cm^2)	E_{TH}/E_p (eV)	$B^3\Pi_g$† ($\times 10^{-17}$ cm^2)	E_{TH}/E_p (eV)	$A^3\Sigma_u^+$‡ ($\times 10^{-17}$ cm^2)	E_{TH}/E_p (eV)	$A^2\Pi_u$† ($\times 10^{-17}$ cm^2)	E_{TH}/E_p (eV)	$B^2\Sigma_u^+$§ ($\times 10^{-17}$ cm^2)
0	11·0/14·7	2·1	7.4/10·2	0.73	6·2/9·1	0·017	16·7/100	3·0	18·8/100	2·48
1	11·3/15·0	1·15	7·6/10·4	1.8	6·3/9·2	0·094	16·9/100	3·8	19·0/100	0·262
2	11·5/15·2	0·42	7·8/10·6	2·3	6·5/9·4	0·27	17·2/100	2·8	19·3/100	0·0028
3	11·7/15·4	0·12	8·0/10·8	2·3	6·7/9·6	0·54	17·4/100	1·5		
4			8·2/11·0	1·8	6·8/9·7	0·88	17·6/100	0·70		
5			8·4/11·2	1·3	7·0/9·9	1·21	17·8/100	0·29		
6			8·6/11·4	0·78	7·2/10·1	1·47				
7			8·8/11·6	0·46	7·3/10·2	1·65				
8			8·9/11·8	0·26	7·5/10·4	1·70				
9			9·1/12·0	0·14	7·6/10·5	1·65				
10			9·3/12·2	0·070	7·8/10·7	1·53				
11			9·5/12·4	0·036	7·9/10·8	1·36				
12			9·6/12·5	0·018	8·1/11·0	1·17				
13					8·2/11·1	0·98				
ΣQ_v		3·8		12·0		18·0		12·0		2·76

* E_{TH}—threshold energy.
E_P—energy at peak cross-section.
† From Shemansky and Broadfoot (1971a).
‡ Rough estimate by Shemansky and Broadfoot (1971a) from energy loss measurements by Williams and Doering (1969), Brinkmann and Trajmar (1970), relative to $B^3\Pi_g$.
§ From Borst and Zipf (1970) measurements of $N^{2+}1N_{0,0}$ band.

ground based observations of $N_2$1P and N_2^+M emissions (Shemansky and Vallance Jones, 1968) the fluctuations are greater at high altitudes; on rare occasions the N_2^+M system was too weak to be measured relative to the $N_2$1PG, indicating a variation of an order of magnitude—the reverse situation of very weak $N_2$1PG relative to the N_2^+M system, was never observed. This tendency toward dominance of the $N_2$1PG at high altitudes appears to be a general characteristic of virtually all auroral types. Observations from the ground by Omholt (1957), Shemansky and Vallance Jones (1968) and the recent rocket measurements which are compiled here, all suggest a gradual decrease of the average emission rate of the $N_2$1PG with decreasing altitude, relative to either the N_2^+1N or N_2^+M systems. The vertical marks on the curves given in Fig. 1 indicate the range of observed values which were averaged to produce the plotted points. Thus the observations suggest that the spectral index appropriate to an average auroral emission at high altitude in the 170–200 km region would be $n \approx -1{\cdot}7$, and a type-B red aurora at 80 km, say, would typically correspond to $n \approx -1{\cdot}0$.

The plotted average ratio (I2PG/I1N) was obtained from the measurements by Petrie and Small (1952) and Hunten (1955). The two sets of measurements are in very good agreement and fall in the region of $n = -1{\cdot}3$ in the Fig. The average value due to Hunten is based on the average relative emission rates of the $N_2^+1N_{0,0}$ and $N_22PG_{0,2}$ bands. The value due to Petrie and Small is based on the emission rates of virtually all of the bands observable from the ground. The average ratios (I1PG/IM) are due to estimates by Chamberlain (1961), Shemansky and Vallance Jones (1968) and Gattinger and Vallance Jones (1971). The value due to Chamberlain, $n \approx -1{\cdot}3$, is derived from a compilation of a number of earlier measurements. The value due to Shemansky and Vallance Jones, $n \approx -1{\cdot}6$, is based on the average ratio ($I1PG_{4,1}/IM_{3,0}$), and that due to Gattinger and Vallance Jones, $n \approx -1{\cdot}2$, is based on photometric aircraft measurements of $I1PG_{5,2}/IM_{4,1}$. The average ratio $I1PG_{0,0}/I1N_{0,0}$, $n \approx -1{\cdot}4$, was obtained from recent simultaneous scanning spectrometer observations of the $N_21PG_{0,0}$ and $N_2^+1N_{0,0}$ bands (Harrison, 1969).

The average ratios ($I1PG_{5,2}/I1N_{0,0}$) are due to rocket-borne photometric observations on NASA flights 4·163, 4·217, 4·309 and aircraft observations by Gattinger and Vallance Jones. In Fig. 2 we show the excitation efficiencies obtained as functions of altitude on these various rocket flights. In Fig. 3 we also plot the ratio of ($I1PG_{5,2}/I1N_{0,0}$) as a function of altitude. Shown as well in this figure is the ratio of the residual integrated column emission rates above 150 km from some of the flights. Data from portions of flight 4·162 are included for the sake of completeness. However, this was a very unusual aurora with a bimodal altitude profile in $N_2^+1N_{0,0}$ and 5577 Å emission features (Donahue *et al.*, 1968). We do not include the results for the lower portion of that aurora where the N_2^+ band becomes very strong again relative to the first positive (5, 2) band. It is very probable that unusual excitation conditions existed during this aurora.

It is interesting to notice in the rocket results that the efficiency of excitation does not usually decrease rapidly or monotonically with decreasing altitude until the very lowest border of the aurora is reached. This statement holds for all emission features. The effective cross-sections calculated as functions of the spectral index in Fig. 2c display the same general characteristic. Thus the ratios in Fig. 3 do not change rapidly, flight 4·162 excepted. In fact the auroral electron flux on the average becomes more efficient at ionising N_2 than exciting the low energy positive band systems as it penetrates from 150 km down to 100 km, as we have indicated above. The fact that the ratio of integrated emission rates above 150 km is unfavorable to ionization strongly suggests the presence of an appreciable initial flux of soft electrons in most auroras.

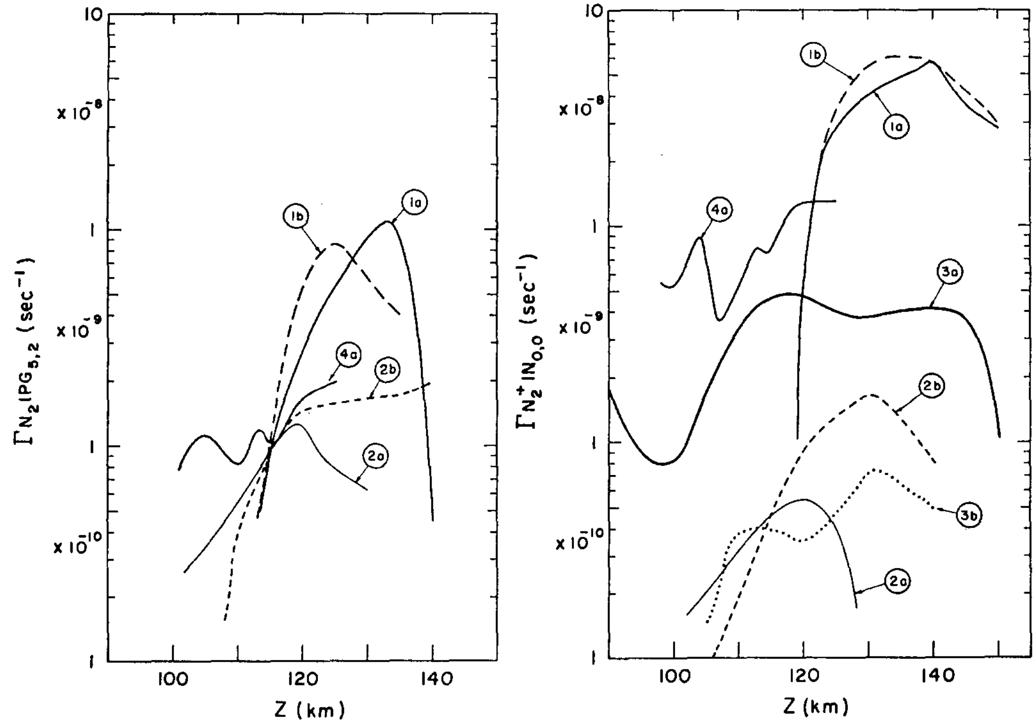

FIG. 2(a, b). EXCITATION EFFICIENCIES OF $N_2 1PG_{5,2}$ AND $N_2^+ 1N_{0,0}$ BANDS.
(1) Flight 4·162 (a) Upleg
(b) Downleg
(2) Flight 4·163
(3) Flight 4·217
(4) Flight 4·309 upleg only.

All of these observations give power law spectral indices in the range $-1{\cdot}4 \leqslant n \leqslant -1{\cdot}2$. We do not include in Fig. 1 the measurements of flight 4·162 due to the unusual, spectacular variations that are not at all representative of normal aurora. The measurements obtained on flight 4·217 were obtained at only two points in the low altitude region due to contamination by moonlight in the $N_2 1PG$ photometer. The results of this flight are included here for the purpose of comparison with low energy electron spectrometer measurements (Feldman *et al.*, 1971) in the same experimental package. It is noteworthy that the relative population rates $g1PG_0/g1PG_5$ obtained from the averaged Harrison and rocket observations are in agreement with the predicted rates.

Thus all of the auroral measurements are consistent with the measured electron excitation cross-sections in that the observed average relative emission rates can be reproduced with a single electron flux distribution function.

The relative intensities of the long and short wavelength ends of the auroral spectrum have always been rather uncertain, for a number of reasons. Photographic spectra were difficult to analyze due to the limited dynamic range and the necessity of comparing features with widely differing differential brightness to brightness ratios. Photoelectric scanning spectrometers were limited in sensitivity and only one set of auroral measurements simultaneously encompassing the N_2^+1N and $N_2 1P$ systems has been published (Hunten, 1955). Extinction of the short wavelength emissions in the lower atmosphere added to the

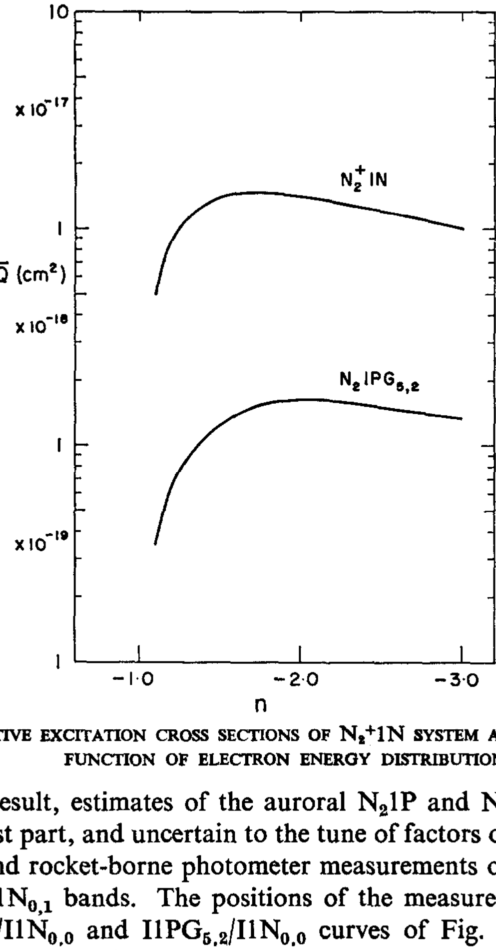

FIG. 2c. EFFECTIVE EXCITATION CROSS SECTIONS OF N_2^+1N SYSTEM AND $N_21PG_{5,2}$ BAND AS A FUNCTION OF ELECTRON ENERGY DISTRIBUTION.*

difficulties. As a result, estimates of the auroral N_21P and N_2^+M brightness have been indirect for the most part, and uncertain to the tune of factors of 2 or 3. However we now have the aircraft and rocket-borne photometer measurements of the $N_21PG_{5,2}$, $N_21PG_{4,1}$, $N_2^+1N_{0,0}$ and $N_2^+1N_{0,1}$ bands. The positions of the measured points on the I2PG/1N, I1PG/IM, $I1PG_{0,0}/I1N_{0,0}$ and $I1PG_{5,2}/I1N_{0,0}$ curves of Fig. 1 clearly conform to the relative emission rates (I1PG/I2PG) predicted from the measured electron excitation cross sections. The N_21PG and N_2^+M emission rates in average aurorae thus appear to be established. The earlier estimates by Chamberlain (1961) and Shemansky and Vallance Jones (1968) are a factor of about 3 too large. The predicted relative population rates of the $C^3\Pi_u$, $B^3\Pi_g$, $A^3\Sigma_u^+$, $A^2\Pi_u$, $B^2\Sigma_u^+$ states in average aurorae ($n = -1{\cdot}37$) are given in Table 2. The total population rates correspond to the auroral brightness (kR) in an IBCI aurora, for the corresponding transitions. The predicted relative rates for the N_2V–K ($A^3\Sigma_u^+ - X^1\Sigma_g^+$) system in the $v' = 0$, 1 levels are about an order of magnitude greater than the ground based measurements (cf. Broadfoot and Hunten, 1964) due to the high radiationless deactivation rate of the $A^3\Sigma_u^+$ state; the measured relative rate $IV\text{–}K_0/I2PG_{0,0} \approx 4{\cdot}8$ at 200 km (Sharp, 1971), where the radiationless deactivation rate is much slower, is in good agreement with the predicted value from Table 2, $IV\text{–}K_0/I2PG_{0,0} = 5{\cdot}9$ (cf. Shemansky *et al.*, 1971).

4. DISCUSSION

An electron energy distribution which would be equivalent to an $E^{-1{\cdot}35}$ power law spectrum in exciting the positive systems of N_2 and the negative and Meinel systems of N_2^+ is

* Differential electron flux $\Phi \alpha E^n$.

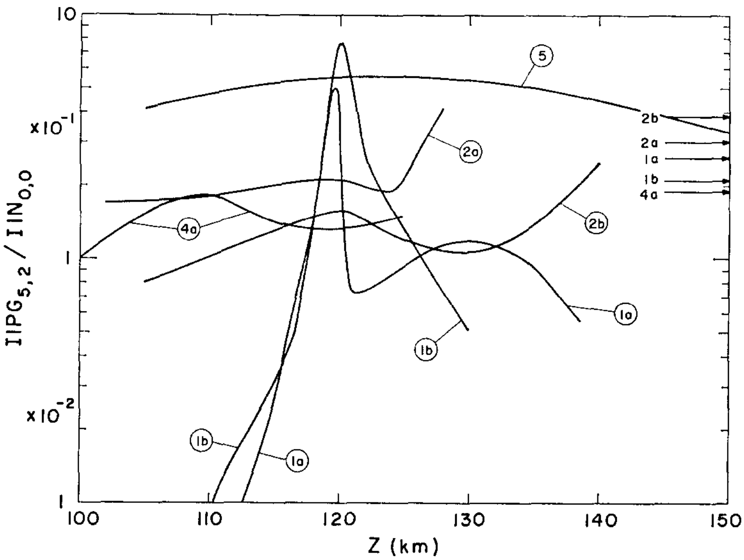

FIG. 3. RATIO OF EXCITATION RATES FOR $N_2 1PG_{5,2}$ BAND TO $N_2^+ 1N_{0,0}$ BAND AS FUNCTION OF ALTITUDE.

(1) Flight 4·162 (a) Upleg
(b) Downleg
(2) Flight 4·163
(3) Flight 4·217
(4) Flight 4·309 upleg only
(5) Calculated from the electron spectra of Rees (1969) and Rees *et al.* (1969).
The labelled horizontal arrows refer to the ratios of integrated emission rates above 150 km.

TABLE 2. PREDICTED RELATIVE POPULATION RATES (g_v) OF N_2 AND N_2^+ STATES IN AVERAGE AURORA

v	$C^3\Pi_u$	$B^3\Pi_g$*	$A^3\Sigma_u^+$†	$A^2\Pi_u$	$B^2\Sigma_u^+$	
0	0·52	0·73	1·50	1·90	1·42	$C^3\Pi_u$—$B^3\Pi_g$—$N_2$2PG
1	0·28	0·98	1·45	2·40	0·150	$B^3\Pi_g$—$A^3\Sigma_u^+$—$N_2$1PG
2	0·10	1·15	1·20	1·77	0·0016	$A^3\Sigma_u^+$—$X^1\Sigma_g^+$—N_2V-K
3	0·029	1·07	1·00	0·95		$A^2\Pi_u$—$X^2\Sigma_g^+$—N_2^+M
4		0·83	0·92	0·44		$B^2\Sigma_u^+$—$X^2\Sigma_g^+$—N_2^+1N
5		0·57	0·92	0·18		
6		0·35	0·93			
7		0·21	0·93			
8		0·11	0·92			
9		0·060	0·87			
10		0·030	0·80			
11		0·015	0·70			
12		0·007	0·59			
13			0·50			
Σg_v‡	0·93	6·1	15·2	7·6	1·58	

* Includes cascade contributions from $N_2$2PG.
† Includes cascade contribution from $N_2$2PG and $N_2$1PG.
‡ Total relative rates represent Brightness in kR in IBCI aurora for the appropriate transitions, provided the populations are controlled entirely by radiative deactivation.

clearly one which changes slope rather dramatically above 50 eV, if the flux decreases as rapidly as E^{-3} at low energies up to 50 eV. In Fig. 4 we have displayed curves showing the differential flux varying as $E^{-1.35}$ (curve 2), raw data of Feldman *et al.* (8), and flux varying as E^{-3} (curve 1). Curve 1 can be taken as the differential flux obtained by Feldman *et al.* after correction of the data (8)—at least out to 50 eV, and also represents the smoothed

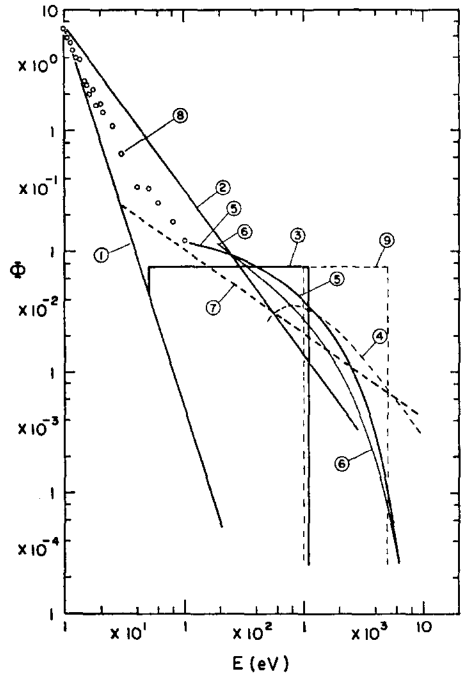

FIG. 4. DIFFERENTIAL ELECTRON FLUX Φ AS A FUNCTION OF ENERGY.

(1) $\Phi \alpha E^{-3}$, arbitrary scale.
(2) $\Phi \alpha E^{-1.37}$, arbitrary scale.
(3) Rectangular electron spectrum $\Delta E = 1$ keV, which in combination with curve 1 is required to produce the observed average auroral relative emission rates.
(4) Rees (1969) primary electron spectrum at 150 km (see text). Secondary spectrum corresponds to curve 1, with the proper relative magnitude.
(5) Stolarski (1968) residual primary spectrum at 150 km (see text), positioned arbitrarily with respect to curve 6.
(6) Stolarski (1968) residual primary spectrum at 120 km (see text).
(7) $\Phi \alpha E^{-0.69}$ the differential flux beginning at 30 eV, which in combination with curve 1 is required to produce the observed average auroral relative emission rates.
(8) Feldman *et al.* (1971) data at 130 km, which after correction, is approximated by curve 1.

theoretical results of Rees *et al.* for secondary electrons. If the (total) flux really follows curve 1 out to 50 eV it would be necessary to extend the flux at its 50 eV value constant as far as 10^{56} eV in order to obtain the observed ratio of the $1N_{0,0}$ band of N_2^+ to the $N_2 1PG_{5,2}$ band. If the effective energy range of the ionizing flux is to be smaller than 10^{56} eV then it is clearly necessary to invoke a discontinuity with an increase in flux somewhere above 50 eV. Because the cross section for excitation of the $N_2^+ 1N_{0,0}$ band peaks at 100 eV, the greater the energy at which the energetic flux maximizes the greater must be the flux at its maximum value. As an example of the requirement, curve 3 shows the magnitude of a constant block of electron flux 1 keV wide starting at 50 eV which would combine with the electrons of curve 1 to give the observed optical ratios. This is an example of a flux spectrum equivalent to a properly normalized $E^{-1\cdot 35}$ power law spectrum. Curve 7 shows the power law variation needed if we assume that the E^{-3} behavior is valid only to 30 eV.

As a matter of fact the combination of curves 1 and 3 would not represent a flux consistent with the rocket spectrometer results of Feldman *et al.* Their spherical analyzer swept in energy up to 800 eV. Their subtraction procedure involves the necessary assumption that the auroral electron flux reaching their analyzer remained less than its value at 50 eV all the way out to 800 eV. Hence the increase in flux as energetic primaries appear would have to come at energies greater than 800 eV—and this at all altitudes below 155 km! A constant flux between 1 keV and 5 keV shown as curve 9 at about the same level as that of curve 3 would have the same ionizing efficiency as the flux of curve 3. Therefore, such a primary spectrum combined with curve 1 would be consistent with the optical data.

The theoretical basis for expecting a gulf in the electron spectrum between 50 eV and several hundred eV, as the combination of curves 1 and 9 would suggest, has been introduced by Rees (1969) and his collaborators (Rees *et al.*, 1969). Rees extended the Maeda (1965) Monte Carlo calculation of the effect of penetration on spreading the energy of an initially monoenergetic electron beam to the case of an initial $E^{-1\cdot 25}$ spectrum (Hoffman, 1969). He predicts a maximum in the keV range, shifting to higher energies with depth of penetration. In Fig. 4 we show as curve 4 the spectrum of degraded primary electrons Rees would predict at 150 km. This curve is properly placed relative to curve 1 if that curve represents the secondary electron distribution of Rees *et al.* at 150 km for the same incident primary electron flux. A large gap, like that demanded by the Feldman *et al.* experimental data as corrected, does exist in this theoretical spectrum. However, the Rees primary flux fails by a factor of about 2 to produce the $N_2^+1N_{0,0}$ excitation rate needed at 150 km (compare curves 4 and 9). Curves 1 and 4 combined are not consistent with the optical data.

This deficiency in ionizing efficiency by the primary electrons points up what appears to be a very serious flaw in the Maeda–Rees degraded primary spectrum. The difficulty is that these electrons ionize the atmosphere at a rate almost a factor of 5 too small to produce the integrated ionization rate inherent in the incident flux. The efficiency of converting electron energy into $N_2^+1N_{0,0}$ band radiation in air is $5\cdot 2 \times 10^{-3}$, independent of electron energy above about 100 eV. This efficiency value is about 20 per cent smaller than the number appropriate to the Rees calculation. However, the $N_2^+1N_{0,0}$ band cross-section in the present calculations is also 20 per cent smaller and the results discussed below should be directly comparable to the Rees case. Thus the column emission rate of $N_2^+1N_{0,0}$ radiation expected from an incident flux of $\phi(E_0)$ electrons cm^{-2} sec^{-1} keV^{-1} sr^{-1} in air is given by

$$\int I1N_{0,0}\,dZ = 1\cdot 6\Omega \int \Phi(E_0)E_0\,dE_0 \tag{6}$$

where Ω is the effective angular distribution of the electrons. The case worked out by Rees is that of a spectrum of primaries observed by Hoffman (1969).

$$\phi(E_0) = 1\cdot 3 \times 10^7 E_0^{-1\cdot 25}\ \mathrm{cm^{-2}\,sec^{-1}\,keV^{-1}\,sr^{-1}}, \quad 0\cdot 7 < E_0 < 25\ \mathrm{keV}. \tag{7}$$

These electrons ought to produce $3\cdot 06 \times 10^8\,\Omega N_2^+1N_{0,0}$ photons cm^{-2} sec^{-1} in air before they are stopped, or 0·97 kR if Ω is π sr.

If we now take the spectra of degraded primaries given by Rees in his 1969 review paper at various altitudes we can compute the local production rate of $N_2^+1N_{0,0}$ photons by evaluating the expression (4):

$$I1N_{0,0} = \Omega[N_2X]\int (Q1N)_{0,0}\Phi(E)\,dE \tag{8}$$

where we use the same cross section (Borst and Zipf, 1970) for producing $N_2^+1N_{0,0}$ radiation that we used to determine the total energy transformation efficiency. This computation gives the following excitation rates at the altitudes indicated:

105 km	33Ω cm^{-3} sec^{-1} (maximum)
110 km	26Ω
120 km	$7{\cdot}4\Omega$
150 km	$2{\cdot}6\Omega$

The integrated rate is $6{\cdot}45 \times 10^7\Omega$ ph cm^{-2} sec^{-1} (or 0·2 kR if Ω is π sr)—a factor of 4·8 less than expected.

Virtually the same result is obtained—and for the same reason—if this $N_2^+1N_{0,0}$ height emission profile is compared with that obtained by Rees from the ionization rate, where the rate is evaluated by using experimental energy dissipation functions and the given incident spectrum. The ratio of ionization rates computed in these two different ways decreases from 0·47 at 150 km to about 0·3 at 105 km where maximum ionization is occurring. By the same token, since the secondary flux is determined by the ionization rate (computed from the dissipation functions and range) these ratios also represent the defect in the predicted ratio, $I1PG_{5,2}/I1N_{0,0}$, compared to the experimental value of about 0·17. The ratios obtained, using the Rees primary spectra are plotted in Fig. 3 (curve 5). They clearly lie very high. The altitude variation of the $I1PG_{5,2}/I1N_{0,0}$ ratio is also divergent with observation. The altitude dependence of curve 5 is presumably due entirely to atmospheric compositional variation, since the primary and secondary spectra are completely decoupled.

It seems, therefore, that there are significant quantities of degraded primaries missing from the predicted spectra. A glance at Fig. 9 in Rees' (1969) review where the spectra are displayed shows that there is room for additional flux only on the low energy side of the maxima in the theoretical spectra. Because of the limited energy range available it seems clear also that to increase ionization rates by factors running from 2 to 5 would require the addition of very large numbers of electrons generally in the range below 5 keV. It is very unlikely that the primary flux near 800 eV will be as small at all altitudes as it would need to be if the Feldman *et al.* (1971) corrected spectrum should be accepted.

A word might therefore be in order regarding the procedure used by Feldman *et al.* (1971) to correct their counting rates for a supposed contribution of photoelectrons from auroral ultraviolet photons reaching their electron multiplier. There is really no experimental evidence to tie the high energy counting rate to this source. The analyzer might well be measuring electron flux validly out to 800 eV. Figure 3 clearly shows that a continuation of the flux (8) at a constant level from 100 eV followed by a primary spectrum such as that calculated by Stolarski would lead to an overall electron spectrum with an effect like the $E^{-1{\cdot}35}$ power law. That is to say that it would be a spectrum with optical excitation properties in agreement with observation. If we suppose that the Rees *et al.* (1969) secondary profiles are valid, this result would call for the presence of considerable fluxes of slow primaries below 100 eV just at the end of their range at all altitudes. It seems to us that the problem of describing the population of slow residual primary auroral electrons is worthy of further attention. We have no reason to question the results of Stolarski, but they cannot describe the primaries below 100 eV. It is not yet at all obvious that the flux at these energies is small compared to the secondary flux in most auroras.

The deficiencies in the spectra of auroral primaries presented by Rees (1969) appear to be inherent in the Monte Carlo results for monoenergetic incident electrons derived by Maeda, rather than in the procedure used by Rees to impose an initial energy variation. Carrying out the comparison of the $N_2^+1N_{0,0}$ band excitation rate expected for 20 keV primaries with that produced by Maeda's published spectra reveals a factor of 9 deficiency there. Twenty kilo-electron volt electrons should produce $1{\cdot}6 \times 20$ or 32 photons per incident electron in air. Applying Equation 6 to the curves in Maeda's Figure yields a maximum excitation rate of $4{\cdot}3 \times 10^{-6}$ cm^{-3} sec^{-1} per incident electron at 97·2 km and an integrated rate of 3·7 cm^{-2} sec^{-1} per electron.

5. CONCLUSIONS

The auroral characteristics of the N_2 positive and N_2^+ systems appear to be well established and consistent with the measured electron excitation cross-sections. These characteristics can be used to place fairly well defined limits on the shape of the auroral electron spectrum. Recent theoretical calculations (Rees, 1969) and direct experimental observations (Feldman *et al.*, 1971) are not compatible with the optical observations according to the present analysis. The disagreement is fundamental in nature and arises in two related but separable phenomena: (1) The Rees theoretical electron spectra are characterized by an energy dependence, $\Phi \alpha E^{-3}$, invariant at low energies, and a high energy spectrum with a relatively sharp peak which moves to higher energies with decreasing altitude. The high and low energy spectra are separated by a deep chasm between 100 eV and the low keV region. This spectral characteristic is in agreement with the experimental auroral estimate at low electron energy (Feldman *et al.*, 1971). However, this electron spectral shape immediately leads to disagreement with the known ionization efficiency for electron impact—a fundamental quantity on which the theoretical calculations must be based. The theoretical spectra obtained by Rees (1969) from the Maeda (1965) Monte Carlo results are seriously deficient in low energy electrons. They do *not* produce rates of excitation of N_2^+ band systems compatible with the known excitation efficiency such as are observed in aurorae. The presence of these slow primaries or fast secondaries would destroy the apparent agreement between the measured differential flux of slow auroral electrons and theoretical spectra, an agreement now holding between these experimental results and calculated secondary spectra. (2) The relative emission rate ratios IN_2/IN_2^+ calculated from the theoretical (Rees, 1969) electron spectra are too large to be compatible with auroral observation. The observed altitude dependence, in which the N_2 emissions dominate at high altitudes is also at variance with the theoretical spectra. The theoretical calculations of Stolarski (1968) on the other hand, do produce the observed IN_2/IN_2^+ ratios, which correspond to a total electron spectrum with $\Phi \alpha E^{-1\cdot 4}$ on the average. The Stolarski calculations appear not to contain the same deficiencies of low energy electrons, although we do not have a direct measure of the electron spectrum below 100 eV in this case.

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